On the Application of Phasor Measurement Units to Power System Stability Monitoring and Analysis

Jiangmeng Zhang and Daniel Chen
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL, USA

***Abstract*—This paper proposes a method for power system stability monitoring and analysis through Phasor Measurement Units' (PMUs) measurements (i.e., bus voltage magnitudes and angles). Real-time stability monitoring can be achieved by observing the behavior of the measurement data's frequency spectra. In addition, a modified Thevenin equivalent-based model is proposed to analyze the system small-signal stability and transient stability. The model's parameters are estimated based on the phasor measurements. We verify our proposed method through simulations on a RTDS/PMU/PDC testbed and perform analyses on real PMU data.**

I. Introduction

As the current power system is operated closer to its physical limits, stability is of high concern for operators. Approaches to assessing power system stability include monitoring and off-line analysis. Real-time stability monitoring refers to a passive approach that observes the behavior of the power system oscillations. For example, a growing system oscillation may suggest instability in the power system, such as the case that caused August 10, 1996 WECC blackout [1]. If the system instability warning is triggered, remedial action should be taken promptly to mitigate the instability. Off-line stability analysis can be used as a preventive approach that identifies the system stability margin under normal operation and predicts the system performance if disturbance occurs. Stability analysis typically include small-signal analysis, which is concerned with how the power system responds to small disturbances, and transient stability analysis, which deals with system response to large disturbances. A diagram showing the approaches to power system stability problem is given in Fig. 1.

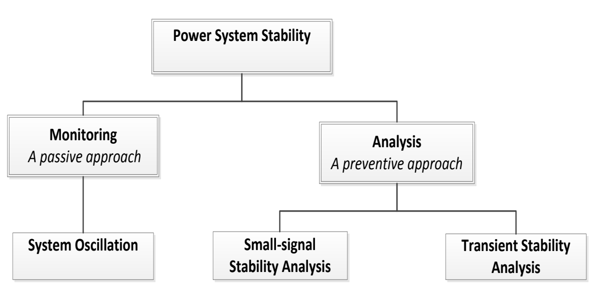
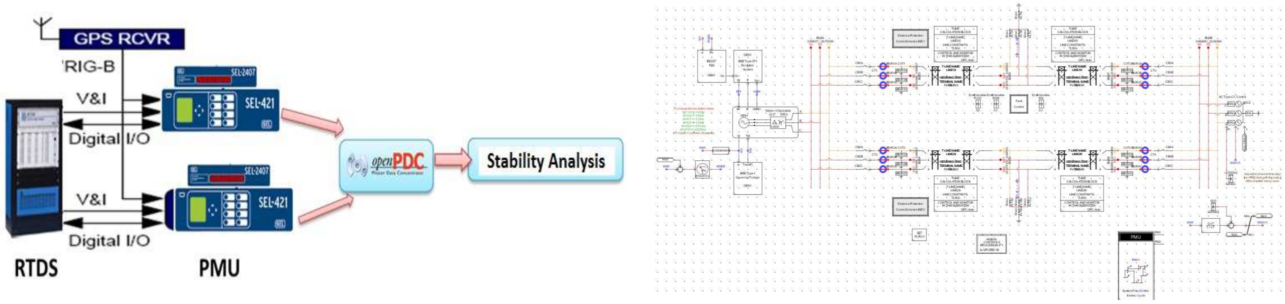


Fig. 1: Power System Stability Assessment Approaches.

The implementation of phasor measurement units across the power system allows for real-time stability monitoring and analysis as they provide synchronized measurements of the bus voltage and current phasors. The advantages of PMU measurements over traditional SCADA measurements include: *i*) higher sampling frequency, and *ii*) the ability to provide direct measurement of power system states (i.e., the voltage magnitude and angle of each bus). The high sampling frequency enables PMUs to capture system changes in a much smaller timescale, allowing for more accurate monitoring of power systems and faster remedial actions. Unlike traditional SCADA system, which estimates the system states, PMUs provide direct measurement of the system states, which can be used to analyze system stability.

Several works have been proposed in applying Digital Signal Processing methods to system stability monitoring. For example, the Prony method and its variations, which expresses PMU measurements as a linear combination of damping sinusoids, are discussed in [2], [3]. Matrix Pencil method and Hankel Total Least Square method also have been proposed to monitor system oscillation [4], [3], [5]. All these monitoring methods tend to capture detailed information of the system performance, which not only require more computation time but also may be more than necessary in emergency situations, where actions need to be taken quickly. In regards to power system stability analysis, the authors in [6] proposed the use of Thevenin equivalents obtained from PMU phasor measurements. However, the model requires that the original equivalent system to be in steady-state, which limits its applications. This method has been applied to load voltage stability analyses, where the Thevenin equivalent is assumed to be constant compared to the change in load, but proves inadequate in stability analysis of general transmission systems. Another measurement-based framework using dynamic equivalent model was proposed in [7], [8]. In this method, the equivalent parameters are estimated only using the voltage magnitude. All the angle information from PMU measurements is not utilized. The estimated results are sensitive to the noise since the oscillation voltage values are very small. Incidentally, proper filtering is required for this method, resulting in longer computation time.

In this paper, we propose an efficient method that provides significant information on system stability in real-time through making the most of the characteristics of PMU



(a) RTDS/PMU/PDC testbed configuration.

(b) Power system oneline diagram (provided by RTDS Technologies Inc.).

Fig. 2: RTDS simulated power system.

measurements. Our proposed monitoring method attempts to achieve a balance between accuracy and computation time; the Fast Fourier Transform (FFT) algorithm, which is used to calculate Discrete Fourier Transform (DFT), provides a good option to perform real-time stability monitoring. In regards to stability analysis, the Thevenin equivalent-based method we propose in this paper models the Thevenin equivalent source with a dynamic model instead of static source to address the changes in the original power system. The phasor information directly measured by PMUs is used to estimate the model's parameters. Our proposed methods of *real-time stability monitoring* and *measurement-based stability analysis* are validated through simulations in a Real Time Digital Simulator (RTDS)/PMU/Phasor Data Concentrator (PDC) testbed in addition to real PMU data.

The remainder of the paper is organized as follows. A real-time stability monitoring method based on phasor measurements is proposed in Section II. In section III, phasor measurements are utilized to obtain the Thevenin equivalent of power system. Using this model, we analyze the system and identify the stabiliy margin. A real power system case study is conducted in Section IV, and concluding remarks are made in Section V.

II. Stability Monitoring

We propose a measurement-based and model-less approach to observe oscillatory instability in real time. In this section, FFT is applied to monitor the PMU measurements to provide a better understanding about the stability of the power system.

A. Preliminaries: Frequency Domain Analysis and Frequency Spectra

From DFT analysis, the frequency domain of a discrete signal can be expressed as

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j(2\pi/N)kn}, \quad 0\leq n\leq N-1. \tag{1}$$

The magnitude of $X(k)$ is the oscillation amplitude at frequency $\frac{kS}{N}$ $(0\leq k<\frac{N}{2})$, where S is the PMU's sampling rate. Therefore $X(k)$ can provide the frequency spectra of the measurements. The FFT, as an efficient algorithm to compute the DFT, allows for almost real time monitoring of the measurements in frequency domain.

B. Power System Oscillation Monitoring Based on Frequency Spectra

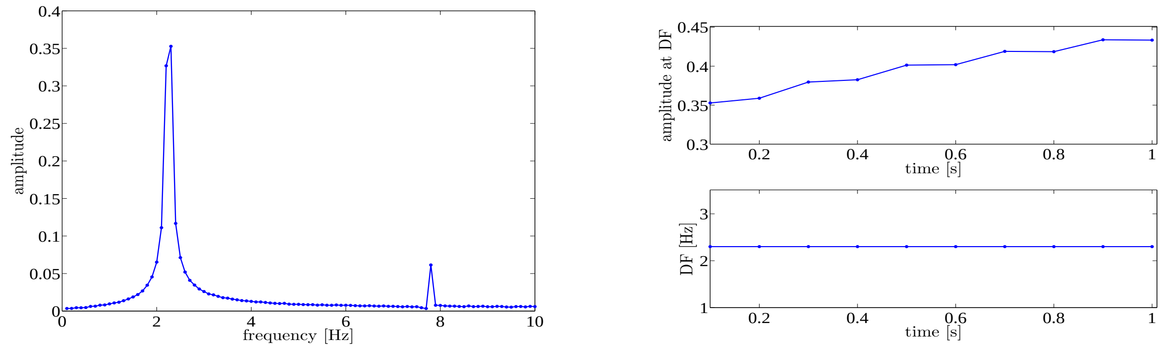
We apply the FFT algorithm to obtain the frequency spectra of the PMU measurement data. The observables in power system (e.g., frequency, phasor magnitude and phasor angle) vary mainly at a same oscillation frequency as a disturbance occurs. This frequency is called as dominant frequency (*DF*). The oscillation amplitude at dominant frequency can be used as a system variable to monitor the system real-time stability. In the frequency domain, if the amplitude at dominant frequency increases without bounds, it is an indication of instability.

C. Experimental Validation

To verify this method, a system with one generator and one load is simulated in a RTDS/PMU/PDC testbed. The testbed configuration and the system one-line diagram are presented in Fig. 2. As shown in Fig. 2(a), the system is simulated in RTDS; the voltage and current measurements on each bus are sent out to one PMU. Then, the phasor data obtained from PMUs are collected in PDC for analysis. In this case, the load is varied by 1% to create a disturbance. FFT analysis is applied to the phasor measurements as the disturbance occurs. The frequency spectra of PMU measurements for the first 200 sample points as the disturbance occurs is shown in Fig. 3(a), from which we can see that the dominant frequency is 2.3 Hz

TABLE I: Thevenin Equivalent Parameter Estimations

	E_1 [p.u.]	E_2 [p.u.]	$\delta_1-\delta_2$ [degree]	X_1 [p.u.]	R_1 [p.u.]	X_2 [p.u.]	R_2 [p.u.]
Traditional method	0.8783	1.2891	-3.4712	0.1245	-0.0411	-0.4537	0.0535
Modified method	0.9671	1.0138	71.7282	0.1080	0	0.1349	0



(a) Frequency spectra for the first 200 sample points. (b) Dominant frequency and its magnitude along with time.

Fig. 3: An unstable case stability monitoring.

and the oscillation amplitude corresponding to this frequency is 0.3528 p.u. The dominant frequency and its amplitude are plotted in Fig. 3(b). In this figure, we conclude from the increasing amplitude that the monitored system is unstable and proper protective action is required. Hence we can say that this stability monitoring method based on FFT can capture system instability in real-time. Another interesting observation is that the dominant frequency remains constant as 2.3 Hz all over time, which is investigaged further in Section III-C

III. Stability Analysis

In this section, a Thevenin equivalent-based model is applied to analyze the system small-signal and transient stability. From this model, a new parameter estimation method is proposed by using the PMU measurements.

A. Thevenin Equivalent-based Model

This power system aggregated model is based on the use of Thevenin equivalents, which guarantees that closely coupled power subsystem can be represented as one Thevenin circuit, which is shown in Fig. 4.

1) Preliminary Work: The authors in [6], [9] proposed that the Thevenin equivalent parameters ($\bar{E}, \bar{Z}_{th}$) can be obtained based on Kirchhoff's Voltage Law with PMU measurements through the following equation

$$\bar{E} - \bar{I} \times \bar{Z}_{th} = \bar{V}, \tag{2}$$

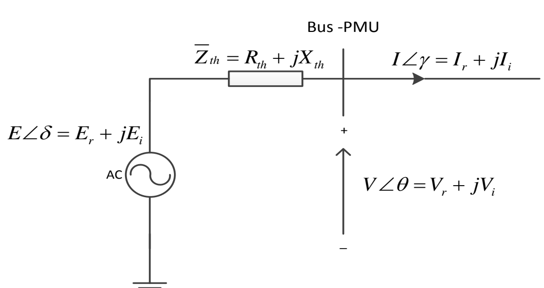


Fig. 4: Thevenin equivalent at terminal bus.

where $\bar{V}$ and $\bar{I}$ are the voltage phasor on the terminal bus and the current phasor following from the terminal bus.

Consider the system in Fig. 4. Let

$$\bar{E} = E\angle\delta = E_r + jE_i, \tag{3}$$

$$\bar{Z}_{th} = R_{th} + jX_{th}. \tag{4}$$

The complex equation (2) can be rewritten as a system of two real-valued equations with four unknown real paramters (i.e., E_r, E_i and R_{th}, X_{th}) as follows,

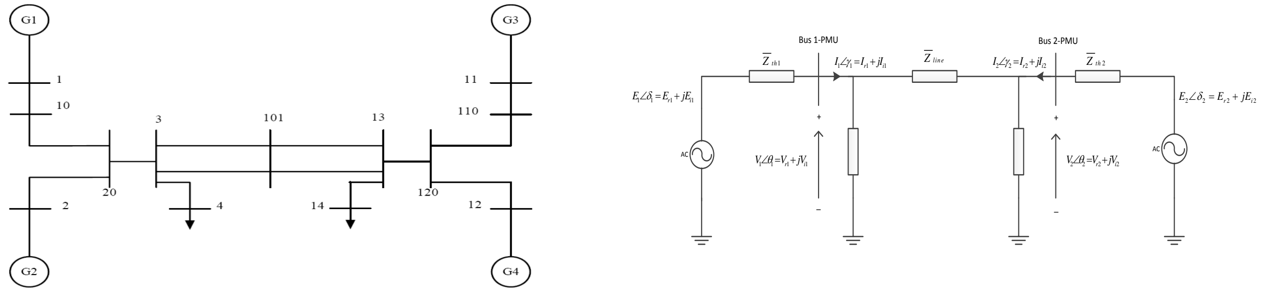
$$\begin{bmatrix} 1 & 0 & -I_r(t) & I_i(t) \\ 0 & 1 & -I_i(t) & -I_r(t) \end{bmatrix} \times \begin{bmatrix} E_r \\ E_i \\ R_{th} \\ X_{th} \end{bmatrix} = \begin{bmatrix} V_r(t) \\ V_i(t) \end{bmatrix}. \tag{5}$$

Assuming that during a short period, the Thevenin equivalent is constant, the measurements of $n(n \geq 2)$ sample points can result $2n$ real-valued equations:

$$\begin{bmatrix} 1 & 0 & -I_r(t_1) & I_i(t_1) \\ 0 & 1 & -I_i(t_1) & -I_r(t_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & -I_r(t_n) & I_i(t_n) \\ 0 & 1 & -I_i(t_n) & -I_r(t_n) \end{bmatrix} \times \begin{bmatrix} E_r \\ E_i \\ R_{th} \\ X_{th} \end{bmatrix} = \begin{bmatrix} V_r(t_1) \\ V_i(t_1) \\ \vdots \\ V_r(t_n) \\ V_i(t_n) \end{bmatrix}, \tag{6}$$

hence the four unknown parameters can be estimated by using least square errors estimation.

2) Modified method: For the former method, the assumption that the Thevenin equivalent keeps constant is acceptable in certain cases, such as in load voltage stability analysis, where Thevenin equivalent is relatively constant compared to the load change. However, for general transmission systems, the changes of the terminal voltage and current might be due to the change of the original power subsystem other than the rest of the power system. Consequently, the Thevenin equivalent needs to be modified to capture the changle of the original subsystem. In the modified approach, the Thevenin equivalent source is represented as the generator's classical model. The angle of Thevenin voltage (i.e., $\bar{E}$) is a time-dependent variable rather than a constant and $R_{th} = 0$ since the resistance is assumed negligible [10]. Based on (2), with



(a) Two-area system.

(b) Two-area system thevenin equivalent.

Fig. 5: Two area system simulation.

PMU measurements at two time points t_1 and t_2, the reactance (X_{th}) equation is derived as follows,

$$\begin{aligned}(I(t_1)^2 - I(t_2)^2)X_{th}^2 + [2I(t_1)V(t_1)\sin(\theta(t_1) - \gamma(t_1)) \\ -2I(t_2)V(t_2)\sin(\theta(t_2) - \gamma(t_2))] \\ + (V(t_1)^2 - V(t_2)^2) = 0. \quad (7)\end{aligned}$$

All the other parameters can be readily found using Kirchhoff's Voltage Law after X_{th} is calculated as the positive root of this quadratic equation.

3) Experimental Validation: To verify the modified method described above, the classical two-area system, as shown in Fig. 5(a), is simulated in MATLAB by using the Power System Toolbox. Assume that two PMUs are set up at the terminals of the inter-area transmission lines to measure the voltage and current phasors on two buses. Based on these measurements, the Thevenin equivalents are obtained on both sides of this transmission line. The power system is reduced to the equivalent system in Fig. 5(b). In this case, the system is simulated to operate at nearby stability limit. Two sets of simulated PMU measurements, $\{\bar{V_1}(t_1), \bar{I_1}(t_1), \bar{V_2}(t_1), \bar{I_2}(t_1)\}$ and $\{\bar{V_1}(t_2), \bar{I_1}(t_2), \bar{V_2}(t_2), \bar{I_2}(t_2)\}$, are collected. The estimation of Thevenin-equivalent parameters by using these two methods are presented in Table I. The table shows that the modified method has a better performance than the tranditional method. For the traditional method, both negative resistance and reactance show up, which is not correct since for a system consisting of positive impedances, the Thevenin equivalent impedance should also be positive. The small angle difference ($\delta_1 - \delta_2$) when operating close to stability limit is not meaningful either; we elaborate on this in detail in Section III-B.

B. Steady-state Stability Analysis

1) Steady-state Stability Measure: Using the Thevenin equivalent-based model, the steady state stability can be readily measured based on the transmission line's loadability (see, e.g., [11], [12], [13]). The angle difference across the system ($\delta_1 - \delta_2$) is called system angle displacement. Since the power flowing into this system is

$$P = \frac{E_1 E_2 \sin(\delta_1 - \delta_2)}{X}, \quad (8)$$

the steady state stability margin (SM) can be measured by a function of system angle displacement as follows,

$$SM = \frac{P_{max} - P}{P_{max}} = 1 - \sin(\delta_1 - \delta_2). \quad (9)$$

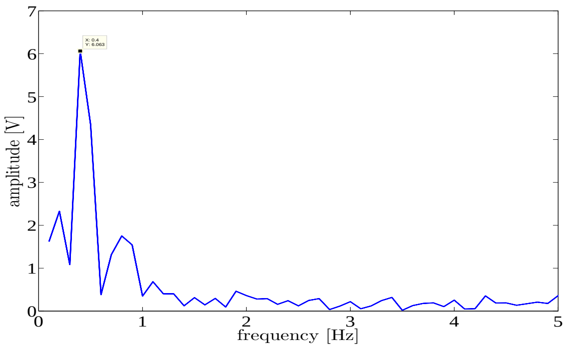
2) Expreimental Validation: For the two-area system example in Section III-A, we can see the system angle displacement is 72^o by using the modified method (see Table I). The SM therefore is 5%, which is reasonable since the system operates near the stability limit. In the traditional method, the system angle displacement is a small negative degree. Consequently, the SM is more than 100%, which does not make sense.

C. Transient Stability Analysis

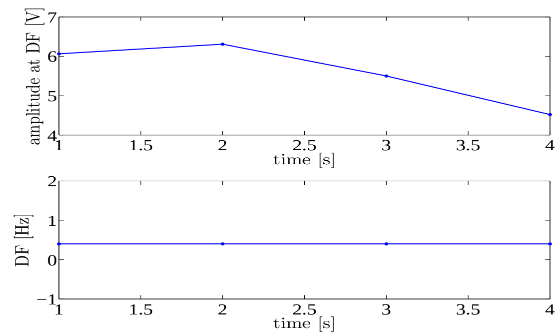
1) Parameter Estimation for Transient Stability Analysis: For power system transient stability analysis, three basic energy function methods (e.g., lowest energy unstable equilibrium point method, potential energy boundary surface method and controlling unstable equilibrium point method) and a number of their variations are discussed in [10]. For the Thevenin equivalent power system, the critical step is to estimate the system parameters in order to apply these energy function methods. Back in Section II, the magnitude of dominant frequency is used to monitor the system oscillation in real time. In this section, the value of the dominant frequency reflects the system's inherent properties. From Fig. 3(b), we observes that the dominant frequency remains constant regardless of the oscillation magnitude. This is due to the fact that this value is determined by the inertia of associated generator or the aggregated equivalent generator. The dynamics of a single machine or an aggregated equivalent machine can be described

TABLE II: Thevenin Equivalent Parameter Estimations

E_1 [p.u.]	1.0508
E_2 [p.u.]	0.8718
X_1 [p.u.]	0.0176
X_2 [p.u.]	0.3645
H [p.u.]	31.8719



(a) Frequency spectra for the first 100 sample points.



(b) Dominant frequency and its magnitude along with time.

Fig. 6: One real case stability monitoring.

using the classical model (see, e.g., [10]) as

$$\begin{aligned} \frac{d\delta}{dt} &= \omega - \omega_s \\ \frac{2H}{\omega_s}\frac{d\omega}{dt} &= T_M - P_e - T_{FW} \\ &= T_M - \frac{EV_s}{X'_d + X_{ep}}\sin(\delta - \theta_{vs}) - T_{FW}. \end{aligned} \tag{10}$$

By linearizing around an operating point, we obtain

$$\begin{aligned} \frac{d}{dt}\Delta\delta &= \Delta\omega \\ \frac{d}{dt}\Delta\omega &= \frac{\omega_s}{2H}[\Delta T_M - \Delta P_e - \Delta T_{FW}]. \end{aligned} \tag{11}$$

Assume that T_M and T_{FW} are constant and $\Delta P_e = \frac{\partial P_e}{\partial \delta}\Delta\delta$. We obtain

$$\frac{d}{dt}\begin{bmatrix}\Delta\delta \\ \Delta\omega\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\omega_s}{2H}\frac{\partial P_e}{\partial \delta} & 0 \end{bmatrix}\begin{bmatrix}\Delta\delta \\ \Delta\omega\end{bmatrix} =: A\begin{bmatrix}\Delta\delta \\ \Delta\omega\end{bmatrix}. \tag{12}$$

The imaginary part of the eigenvalues (λ_1, λ_2) of the matrix A (i.e., $Im\{\lambda_{1,2}\}$) is equal to $\sqrt{\frac{\omega_s}{2H}\frac{\partial P_e}{\partial \delta}}$. It is also proportional to the system oscillation frequency by a factor of 2π. Therefore

$$\sqrt{\frac{\omega_s}{2H}\frac{\partial P_e}{\partial \delta}} = 2\pi f. \tag{13}$$

Notice that we can use the value of system frequency to estimate the generator inertia H as

$$H = \frac{\omega_s}{8\pi^2 f^2}\frac{\partial P_e}{\partial \delta} = \frac{\omega_s}{8\pi^2 f^2}\frac{EV_s}{X'_d + X_{ep}}\cos(\delta - \theta_{vs}). \tag{14}$$

After the power system model and parameters are obtained, stability analysis (e.g., energy function method for transient analysis) can be easily performed [14].

2) Experimental Validation: For the single-machine example in Section II-B, the dominant frequency is 2.3 Hz. The estimated generator inertia H (1.713 p.u.) matches the real inertia value of the generator model set up in RTDS (1.625 p.u.). A more general and important application is to estimate the parameters of the Thevenin equivalent model for the real power system, which is discussed in more detail in Section IV.

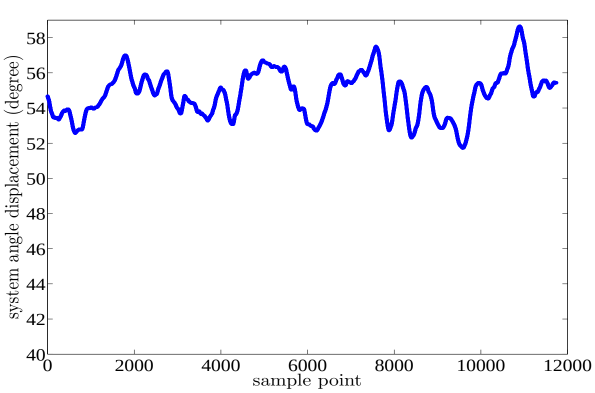


Fig. 7: Real power system thevenin equivalent's system angle displacement.

IV. Real Power System Case Study

A. Application to Real Power System Case

Both the system stability monitoring and analysis methods proposed above are applied to the real PMU data.

For system stability monitoring, the frequency spectra of the measurements at first 100 sample points is shown in Fig. 6(a). During the following four seconds, the FFT results are shown in Fig. 6(b). For stability monitoring, the oscillation amplitude at the dominat frequency decreases with time, which verifies that the system is stable during this period. This system's dominant frequency is 0.4 Hz, which, as expected, remains constant. This value is also a critical parameter for stability analysis.

For the system stability analysis, with phasor measurements on two terminals of the transmission line, two Thevenin equivalents are obtained on both sides of the transmission line. The power system network is reduced to Fig. 5(b). The estimated parameters for Thevenin voltage and impedance are presented in Table II. The time variable (i.e., system angle displacement $\delta_1 - \delta_2$) is plotted in Fig. 7. The system SM can be calculated by taking the average for a period of interest as follows,

$$SM = \frac{1}{N} \sum_{i=1}^{N} (1 - \sin(\delta_1[i] - \delta_2[i])) \tag{15}$$

where N is the total number of measurements in the period. Using (15), the SM of this system is computed as 23.4%.

Along with the dominant frequency obtained in stability monitoring section, the system equivalent inertia is readily estimated using equation (14). The result for this power system is also listed in Table. II. The inertia value (31.8719 p.u.) is close to the inertia value of the RTDS system simulated at the same oscillation frequency in Section III.

V. Conclusion

In this paper, a comprehensive framework for applying PMU data to power system stability monitoring and analysis is proposed. In this framework, frequency analysis of high resolution PMU data allows the operators to monitor the system stability more accurately. For stability analysis, to reduce the system scale, a system Thevenin equivalent model as a power system aggregation and reduction method is presented. Steady state stability can be measured based on this Thevenin equivalent. With the utilization of dominant frequency from frequency analysis, all the parameters of the equivalent model are estimated for transient stability analysis. These methods are verified in simulations on a RTDS/PMU/PDC testbed case and in addition through real PMU data.

References

[1] D. Kosterev, C. Taylor, and W. Mittelstadt, "Model validation for the august 10, 1996 wscc system outage," *Power Systems, IEEE Transactions on*, vol. 14, no. 3, pp. 967 –979, aug. 1999.

[2] S. E. L. Inc, "Synchrophasor-based online modal analysis to mitigate power system interarea oscillation."

[3] G. Liu, V. Venkatasubramanian, and J. Carroll, "Oscillation monitoring system using synchrophasors," in *Power Energy Society General Meeting, 2009. PES '09. IEEE*, july 2009, pp. 1 –4.

[4] A. Meliopoulos, G. Cokkinides, O. Wasynczuk, E. Coyle, M. Bell, C. Hoffmann, C. Nita-Rotaru, T. Downar, L. Tsoukalas, and R. Gao, "PMU data characterization and application to stability monitoring," in *Power Systems Conference and Exposition, 2006. PSCE '06. 2006 IEEE PES*, 29 2006-nov. 1 2006, pp. 151 –158.

[5] J. Wang, R. Gardner, and Y. Liu, "Analysis of system oscillations using wide-area measurements," in *Power Engineering Society General Meeting, 2006. IEEE*, 0-0 2006, p. 6 pp.

[6] K. Vu, M. Begovic, D. Novosel, and M. Saha, "Use of local measurements to estimate voltage-stability margin," *Power Systems, IEEE Transactions on*, vol. 14, no. 3, pp. 1029 –1035, aug. 1999.

[7] J. Chow, A. Chakrabortty, L. Vanfretti, and M. Arcak, "Estimation of radial power system transfer path dynamic parameters using synchronized phasor data," *Power Systems, IEEE Transactions on*, vol. 23, no. 2, pp. 564 –571, may 2008.

[8] A. Chakrabortty, J. Chow, and A. Salazar, "A measurement-based framework for dynamic equivalencing of large power systems using WAMS," in *Innovative Smart Grid Technologies (ISGT), 2010*, jan. 2010, pp. 1 –8.

[9] Y. Song, "Design of secondary voltage and stability controls with multiple control objectives," dec. 2008.

[10] P. Sauer and M. Pai, *Power System Dynamics and Stability*. Upper Saddle Rever, NJ: Prentice-Hall, Inc., 1998.

[11] H. P. S. Clair, "Practical concepts in capability and performance of transmission lines," *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, vol. 72, no. 2, pp. 1152 –1157, jan. 1953.

[12] R. Gutman, "Application of line loadability concepts to operating studies," *Power Systems, IEEE Transactions on*, vol. 3, no. 4, pp. 1426 –1433, nov. 1988.

[13] R. Gutman, P. Marchenko, and R. Dunlop, "Analytical development of loadability characteristics for EHV and UHV transmission lines," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-98, no. 2, pp. 606 –617, mar. 1979.

[14] A. Fouad and V. Vittal, "Power system response to a large disturbance: Energy associated with system separation," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-102, no. 11, pp. 3534 –3540, nov. 1983.