Static State Estimation in Power Systems

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Energy Management System (EMS)

1. Measurements
   - Obtained from supervisory control and data acquisition (SCADA)
   - New data every 4 seconds
2. Static state estimator
   - Uses a model of the system that includes topology and parameter information
   - Estimates real-time values for the whole system
   - Signals to operators any lines close to overload or already overloaded
   - MISO runs this every 5 minutes

Static State Estimation

- Process to assign values to unknown system state variables based on *imperfect* measurements obtained from the system
- State variables:
  1. Voltage magnitudes
  2. Relative phase angles at buses
- Measurements:
  1. Voltage magnitudes
  2. Real and reactive transmission line flows
  3. Current through transmission lines
- Objective: produce the "best estimate" of the system voltage and phase angles, recognizing that there are errors in the measured quantities and that there may be redundant measurements.
- Basic underlying assumption: power system is in quasi-steady-state condition

Static State Estimation Criteria

- Common criteria for “best” estimate
  1. The maximum likelihood criterion: maximize probability that estimate of the state variable is its true value
  2. The weighted least-squares criterion: minimize sum of squares of the weighted deviations of estimated measurements from actual measurements
  3. The minimum variance criterion: minimize expected value of sum of squares of the the deviations of estimated components of the state vector from the corresponding true values
- Assume meter error distributions are normally distributed and unbiased
- Then the three criteria above lead to identical estimators

Maximum Likelihood Least-Squares Estimation

- Let x denote vector of state variables, i.e., voltage magnitudes and relative phase angles
- Let η_i represent uncertainty in each measured value z_i^{meas}. Then,

$$z_i^{meas} = z_i^{true} + \eta_i = h_i(x) + \eta_i$$

- Model $\eta_i \sim \mathcal{N}(0, \sigma_i)$
- Assumptions:
  1. Zero-mean: $\mathbb{E}[\eta_i] = 0$, for all i
  2. Independent measurements:

$$\mathbb{E}[\eta_i \eta_j] = \begin{cases} \sigma_j^2 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Maximum Likelihood Least-Squares Estimation

- The p.d.f. of z_i^{meas} is

$$f(z_i^{meas}) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left\{ \frac{-\left[z_i^{meas} - h_i(x)\right]^2}{2\sigma_i^2} \right\}$$

- Find estimate $\hat{x}$ that maximizes the probability that the observed measurement z_i^{meas} would occur

  $\Longrightarrow$ Find $\hat{x}$ that maximizes p.d.f. $f(z_i^{meas})$

  $\Longrightarrow$ Find $\hat{x}$ that minimizes $\frac{\left[z_i^{meas} - h_i(x)\right]^2}{2\sigma_i^2}$

- Suppose there are m measurements
- Leads to the following objective:

$$\min J(x) = \sum_{i=1}^{m} \frac{\left[z_i^{meas} - h_i(x)\right]^2}{2\sigma_i^2}$$

Matrix Form

- Let

$$z = \begin{bmatrix} z_1^{meas} \\ \vdots \\ z_m^{meas} \end{bmatrix}, \quad h(x) = \begin{bmatrix} h_1(x) \\ \vdots \\ h_m(x) \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_m^2 \end{bmatrix}$$

- Weighted least-squares (WLS) estimation problem with R^{-1} as weighting matrix

$$\min J(x) = \frac{1}{2} \left[z - h(x)\right]^T R^{-1} \left[z - h(x)\right] = J(\hat{x})$$

- Necessary conditions of optimality are

$$\nabla_x J\Big|_{x=\hat{x}} = -\left[z - h(\hat{x})\right]^T R^{-1} \underbrace{\left[\nabla_x h\right]_{x=\hat{x}}}_{H(\hat{x})} = 0$$

- So $\hat{x}$ is the root of the set of nonlinear equations

$$H^T(x) R^{-1} \left[z - h(x)\right] = 0$$

Weighted Least-Squares Estimation

$$H^T(x)R^{-1}\left[z - h(x)\right] = 0$$

- Can solve iteratively. Or...
- If each h_i is a linear function* of x, i.e., $h(x) = Hx$, then

$$H^T R^{-1} z - H^T R^{-1} H x = 0$$

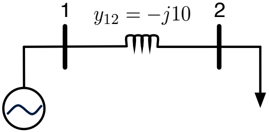
- Closed-form solution exists:

$$\hat{x} = \left[H^T R^{-1} H\right]^{-1} H^T R^{-1} z$$

* DC power flow model assumptions

- For each of the transmission lines, $x_i >> r_i$.
- All bus voltages are approximately 1 p.u.
- All bus angles are nearly 0. Therefore, $\cos(\theta_i - \theta_j) \cong 1$ and $\sin(\theta_i - \theta_j) \cong \theta_i - \theta_j$.

Example—Two-Bus System



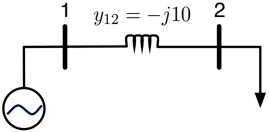
- 4 measurements

$$z = \begin{bmatrix} v_1 \\ v_2 \\ p_{12} \\ q_{12} \end{bmatrix} = \begin{bmatrix} 1.02 \\ 1.0 \\ 2.0 \\ 0.2 \end{bmatrix}, \quad R = \begin{bmatrix} 0.05^2 & 0 & 0 & 0 \\ 0 & 0.05^2 & 0 & 0 \\ 0 & 0 & 0.1^2 & 0 \\ 0 & 0 & 0 & 0.1^2 \end{bmatrix}$$

- 3 states

$$x = \begin{bmatrix} V_1 \\ V_2 \\ \theta_2 \end{bmatrix}$$

Example—Two-Bus System



- Measured values

$$z = h(x) = \begin{bmatrix} v_1 \\ v_2 \\ p_{12} \\ q_{12} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -10V_1V_2\sin\theta_2 \\ 10V_1^2 - 10V_1V_2\cos\theta_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}$$

- Jacobian

$$H(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10V_2\sin\theta_2 & -10V_1\sin\theta_2 & -10V_1V_2\cos\theta_2 \\ 20V_1 - 10V_2\cos\theta_2 & -10V_1\cos\theta_2 & 10V_1V_2\sin\theta_2 \end{bmatrix}$$

- Iterative solution. At the k^{th} iteration, solve for Δx_k in

$$[H(x_k)]^T R^{-1} H(x_k)\,\Delta x_k = [H(x_k)]^T R^{-1}[z - h(x_k)]$$