Optimal Power Flow (OPF) and Security Constrained OPF

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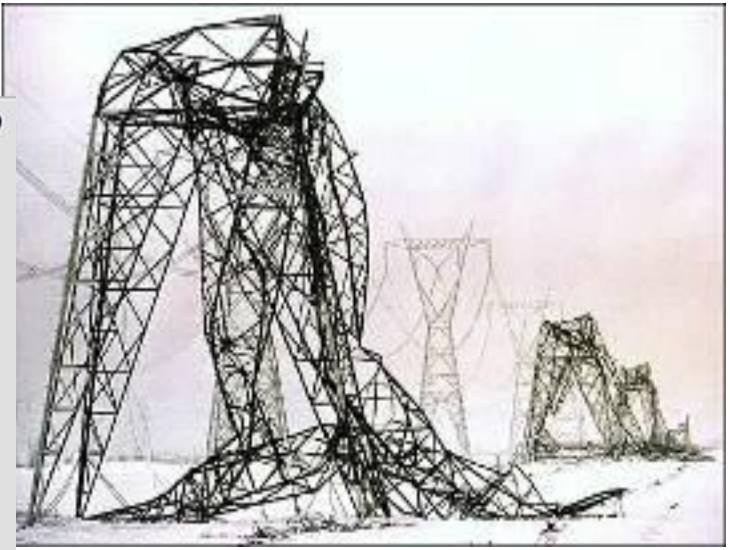
In the News: Ice and Power Lines

Ice and power lines do not mix well!

The Sperry-Piltz Ice Accumulation Index, or "SPIA Index" – Copyright, February, 2009

ICE DAMAGE INDEX	* AVERAGE NWS ICE AMOUNT (in inches) *Revised-October, 2011	WIND (mph)	DAMAGE AND IMPACT DESCRIPTIONS
0	< 0.25	< 15	Minimal risk of damage to exposed utility systems; no alerts or advisories needed for crews, few outages.
1	0.10 – 0.25	15 - 25	Some isolated or localized utility interruptions are possible, typically lasting only a few hours. Roads and bridges may become slick and hazardous.
	0.25 – 0.50	> 15	
2	0.10 – 0.25	25 - 35	Scattered utility interruptions expected, typically lasting 12 to 24 hours. Roads and travel conditions may be extremely hazardous due to ice accumulation.
	0.25 – 0.50	15 - 25	
	0.50 – 0.75	< 15	
3	0.10 – 0.25	>= 35	Numerous utility interruptions with some damage to main feeder lines and equipment expected. Tree limb damage is excessive. Outages lasting 1 – 5 days.
	0.25 – 0.50	25 - 35	
	0.50 – 0.75	15 - 25	
	0.75 – 1.00	< 15	
4	0.25 – 0.50	>= 35	Prolonged & widespread utility interruptions with extensive damage to main distribution feeder lines & some high voltage transmission lines/structures. Outages lasting 5 – 10 days.
	0.50 – 0.75	25 - 35	
	0.75 – 1.00	15 - 25	
	1.00 – 1.50	< 15	
5	0.50 – 0.75	>= 35	Catastrophic damage to entire exposed utility systems, including both distribution and transmission networks. Outages could last several weeks in some areas. Shelters needed.
	0.75 – 1.00	>= 25	
	1.00 – 1.50	>= 15	
	> 1.50	Any	

ategories of damage are based upon combinations of precipitation totals, temperatures and wind speeds/directions.



Quebec Ice Storm, January 1998; Some places got more than three inches of ice!

Image source: http://i.imwx.com/web/spia-index-web.jpg

Overview

- The goal of an optimal power flow (OPF) is to determine the "best" way to instantaneously operate a power system.
  - Usually "best" = minimizing operating cost.
- OPF combines a power flow solution with an economic dispatch solution to determine the best way to operate the system without limit violations
- Security constrained OPF (SCOPF) includes the impacts of contingencies
- OPF is used as basis for real-time pricing in major US electricity markets such as MISO and PJM.

OPF Constraints

- OPF is an optimization that seeks to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Example equality constraints (primarily from the power flow)
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

OPF Constraints, cont'd

- Example Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes
  - transformer taps, phase shifts
- Example Controls
  - generator MW outputs and voltage setpoints
  - transformer taps and phase angles
  - switched shunts
  - loads

OPF Mathematical Formulation

- The OPF can be formulated quite simply as

$$\begin{aligned} &\min \; f(\mathbf{x},\mathbf{u}) \\ &s.t. \quad \mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0} \\ &\qquad\quad \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{0} \end{aligned}$$

where $\mathbf{x}$ is the vector of the state variables, primarily the bus voltage magntiudes and angles, and $\mathbf{u}$ is the vector of controls

- But understanding OPF requires we briefly review power flow and economic dispatch

Power Flow Analysis

- The most common power system analysis tool is the power flow (also known sometimes as the load flow)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a quasi steady-state analysis tool; it is valid on a time period of minutes (load is assumed constant)

Power Flow Analysis

- Power flow involves solving the nonlinear power balance equations to enforce KCL at each bus in the system
  - Net real and reactive power into bus sums to zero
- Inputs into the power flow are a network model (e.g., system connectivity and line impedances), loads being consumed at the buses (PQ buses), real power injected by the generators, and the generator setpoint voltage magnitudes (PV buses); also require a system slack bus

Power Flow: 3-Phase vs Single-Phase

- Power flow (and OPF) are usually solved assuming the underlying three-phase system is balanced, hence an equivalent per phase network is solved (also known as the positive sequence)
- Full three-phase models can be used but they are much more complicated

Power Flow Equations

- Power flow equations can be represented as just a set of algebraic equations; essentially twice the number of buses since there is a real power balance equation and a reactive power balance equation

$$\mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

where $\mathbf{x}$ is the vector of bus voltage angles and magnitudes; $\mathbf{u}$ is a vector of mostly fixed parameters (which will be varied in the OPF)

Actual Power Flow Equations

- At each bus i we have

$$S_i = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^{*} V_k^{*} = \sum_{k=1}^{n} |V_i||V_k| \, e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

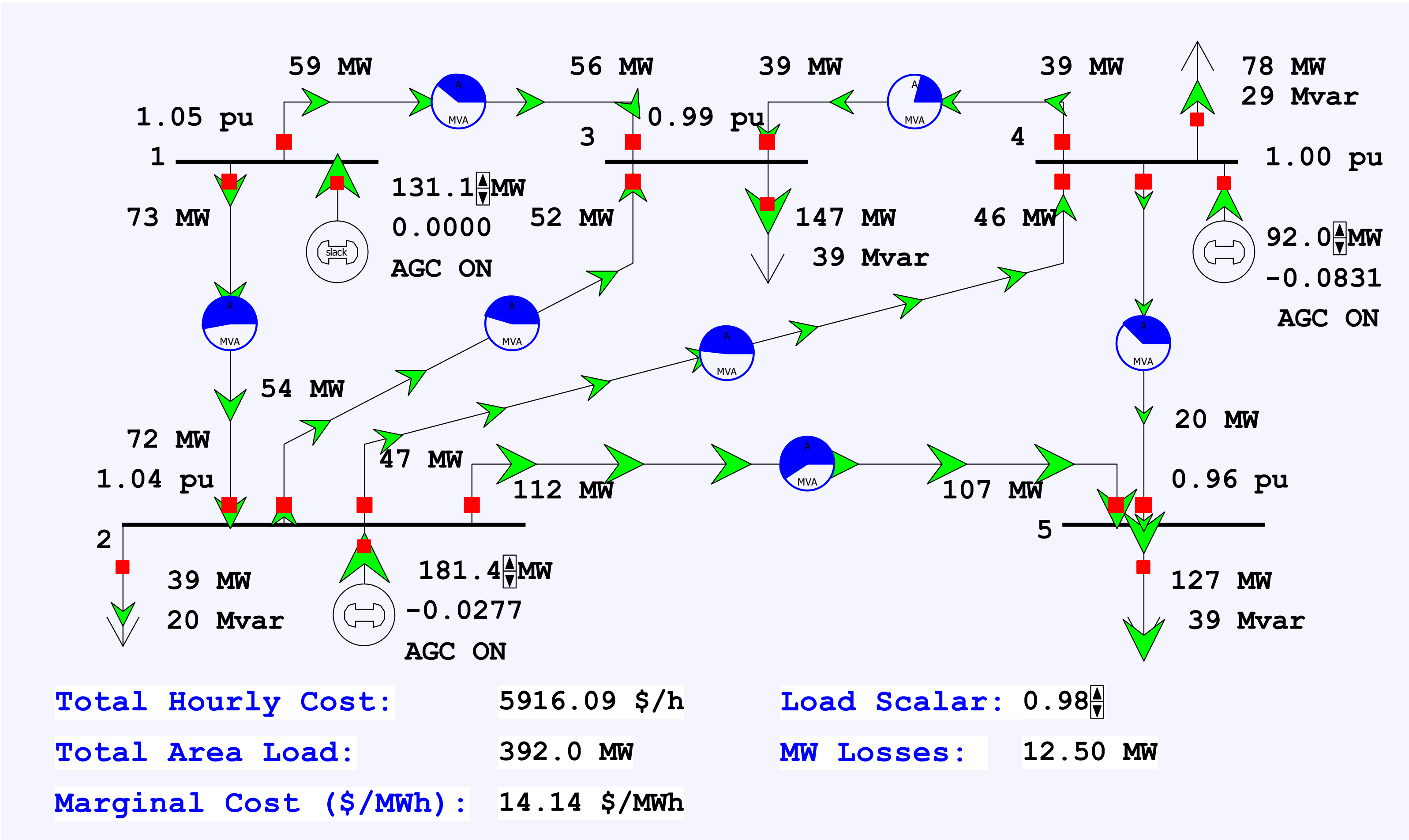
$$= \sum_{k=1}^{n} |V_i||V_k| \, (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

Resolving into the real and imaginary parts

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

Power Flow Example: Five Buses

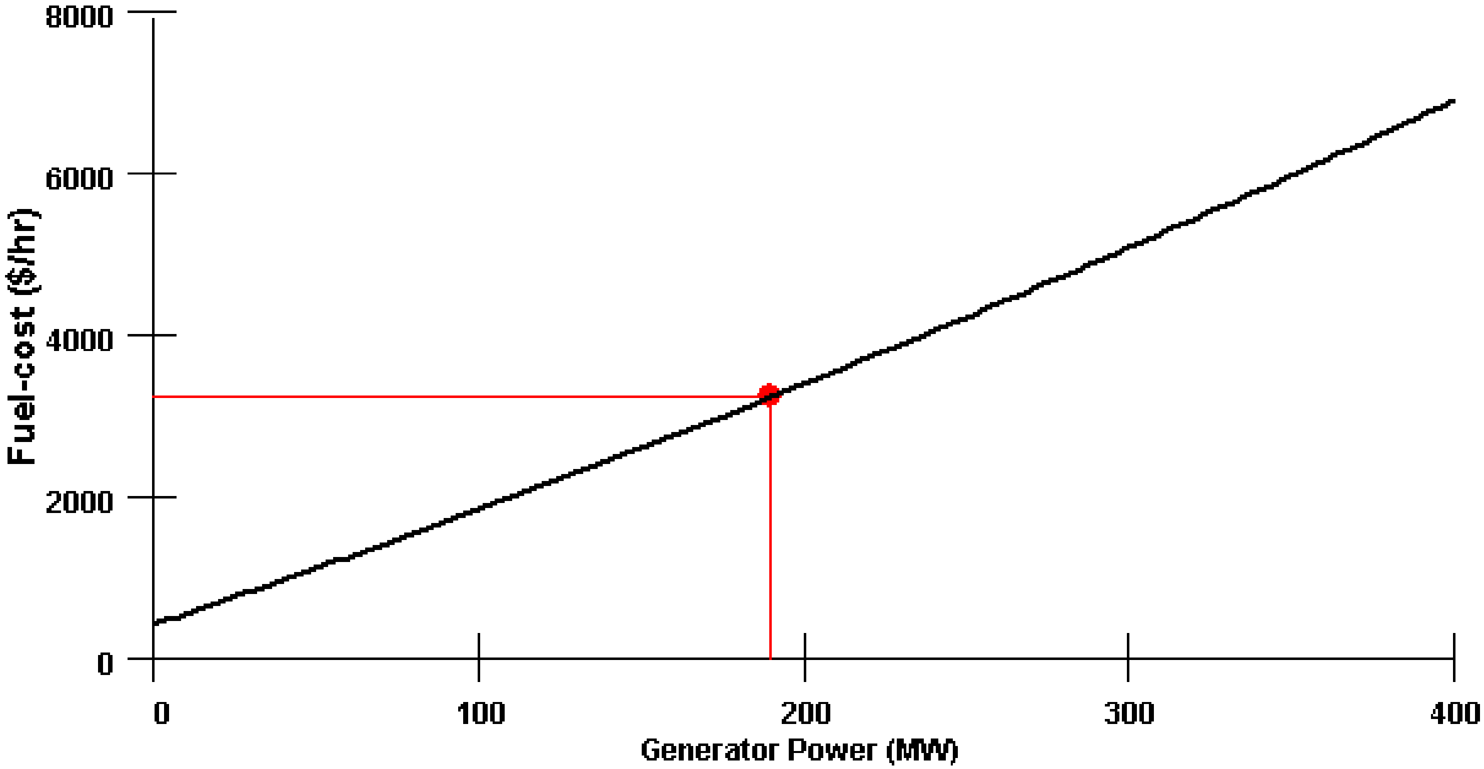


Economic Dispatch

- Different generation resources have different marginal (incremental) cost characteristics
  - Cost to get the next MWh from the resource
  - Capital costs are not considered here
  - Some, such as wind and solar, may have quite low (or sometimes even negative marginal costs); others, such as natural gas or oil, can be much higher
- Economic dispatch seeks to minimize the total generation dispatch costs

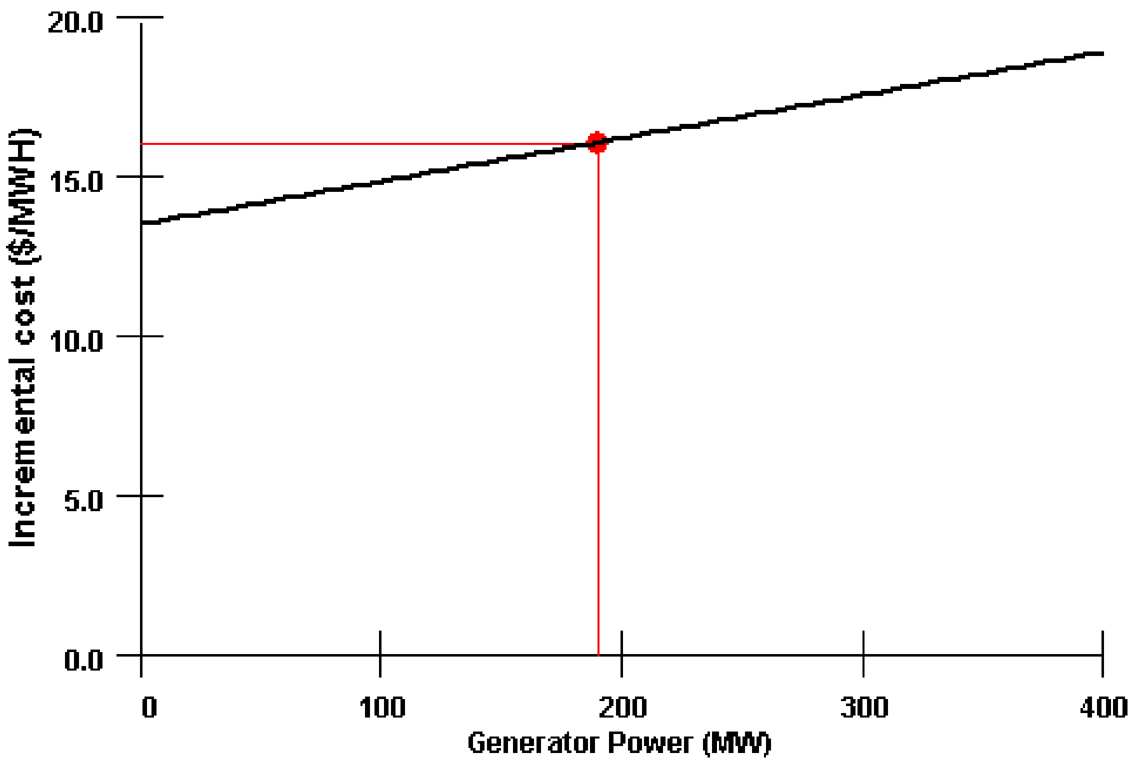
Generator Cost Curve

- The fuel-cost curve shows how the costs of operating a unit vary with its MW output



Incremental (Marginal) cost Curve

- Plots the incremental $/MWh as a function of MW.
- Found by differentiating the cost curve



Economic Dispatch: Formulation

- Economic dispatch can be formulated as a constrained minimization

$$\text{Minimize} \qquad \mathrm{C_T} \ \square \ \sum_{i=1}^{m} C_i(P_{Gi})$$

$$\text{Such that} \qquad \sum_{\mathrm{i=1}}^{\mathrm{m}} P_{Gi} = P_D + P_{Losses}$$

Or more generally as

$$\min \ f(\mathbf{u})$$

$$\begin{aligned} s.t. \quad \mathbf{g}(\mathbf{u}) &= \mathbf{0} \\ \mathbf{h}(\mathbf{u}) &\leq \mathbf{0} \end{aligned}$$

The inequality constraints are introduced to include generator limits

Economic Dispatch and Optimality

- The optimal (or economic) dispatch occurs when all the generators have equal marginal costs, excepting that some generators will be constrained at their maximum or minimums
  - Become of the time constants (hours) associated with starting/shutting down large thermal units, they often run for a while at their minimum values
  - Determining which generators should be online is known as unit commitment, and is not considered here
- In this case the locational marginal price (LMP) would be the same at each bus in the system

Economic Dispatch Example

What is economic dispatch for a two generator system $\mathrm{P_D} = P_{G1} + P_{G2} = 500$ MW and

$$C_1(P_{G1}) \quad = \quad 1000 + 20P_{G1} + 0.01P_{G1}^2 \; \$/hr$$

$$C_2(P_{G2}) \quad = \quad 400 + 15P_{G2} + 0.03P_{G2}^2 \qquad \$/hr$$

Using the Largrange multiplier method we know

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda \qquad = 20 + 0.02P_{G1} - \lambda \qquad = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda \qquad = 15 + 0.06P_{G2} - \lambda \qquad = 0$$

$$500 - P_{G1} - P_{G2} \; = \; 0$$

Economic Dispatch Example, cont'd

We therefore need to solve three linear equations

$$20 + 0.02 P_{G1} - \lambda \qquad = 0$$

$$15 + 0.06 P_{G2} - \lambda \qquad = 0$$

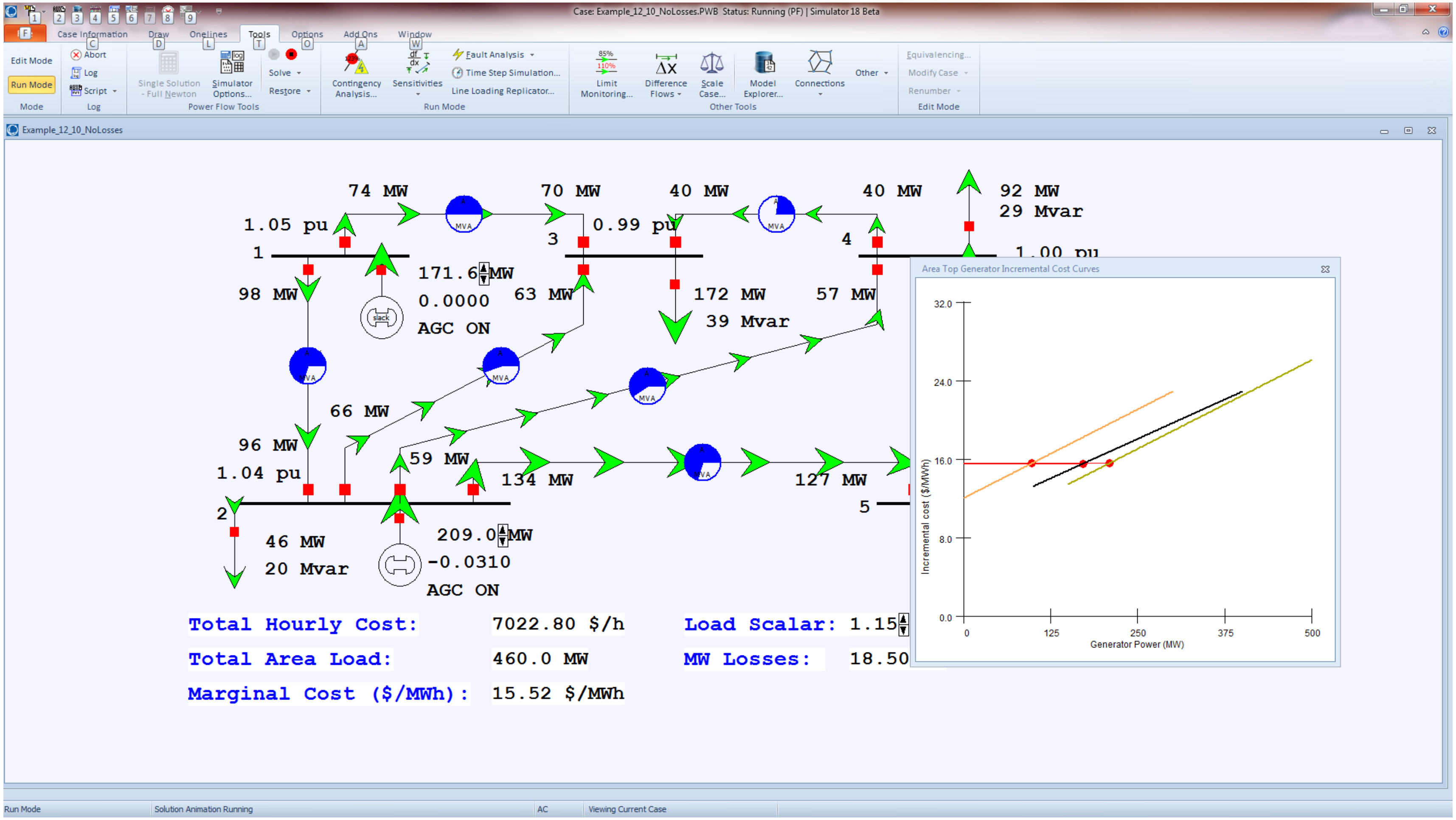
$$500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

The marginal cost, λ, is the incremental cost to supply one more MWh

Five Bus Case Economic Dispatch



Marginal impact of losses is ignored

Inclusion of Transmission Losses

- The losses on the transmission system are a function of the generation dispatch. In general, using generators closer to the load results in lower losses
- This impact on losses should be included when doing the economic dispatch
- Losses can be included by slightly rewriting the economic dispatch Lagrangian:

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

Impact of Transmission Losses

This small change then impacts the necessary conditions for an optimal economic dispatch

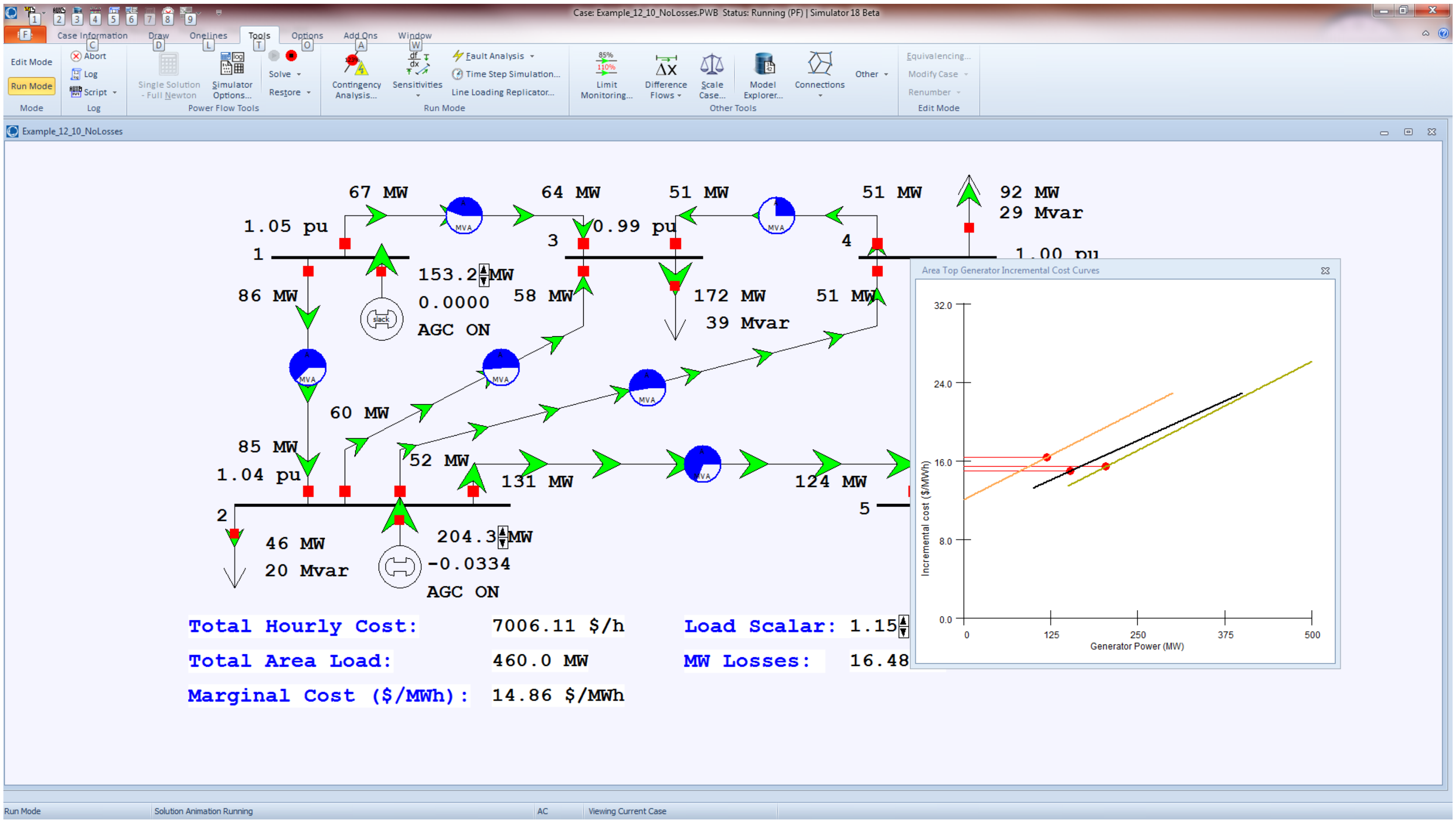
$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

The necessary conditions for a minimum are now

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

Five Bus Case Economic Dispatch



Marginal impact of losses is included

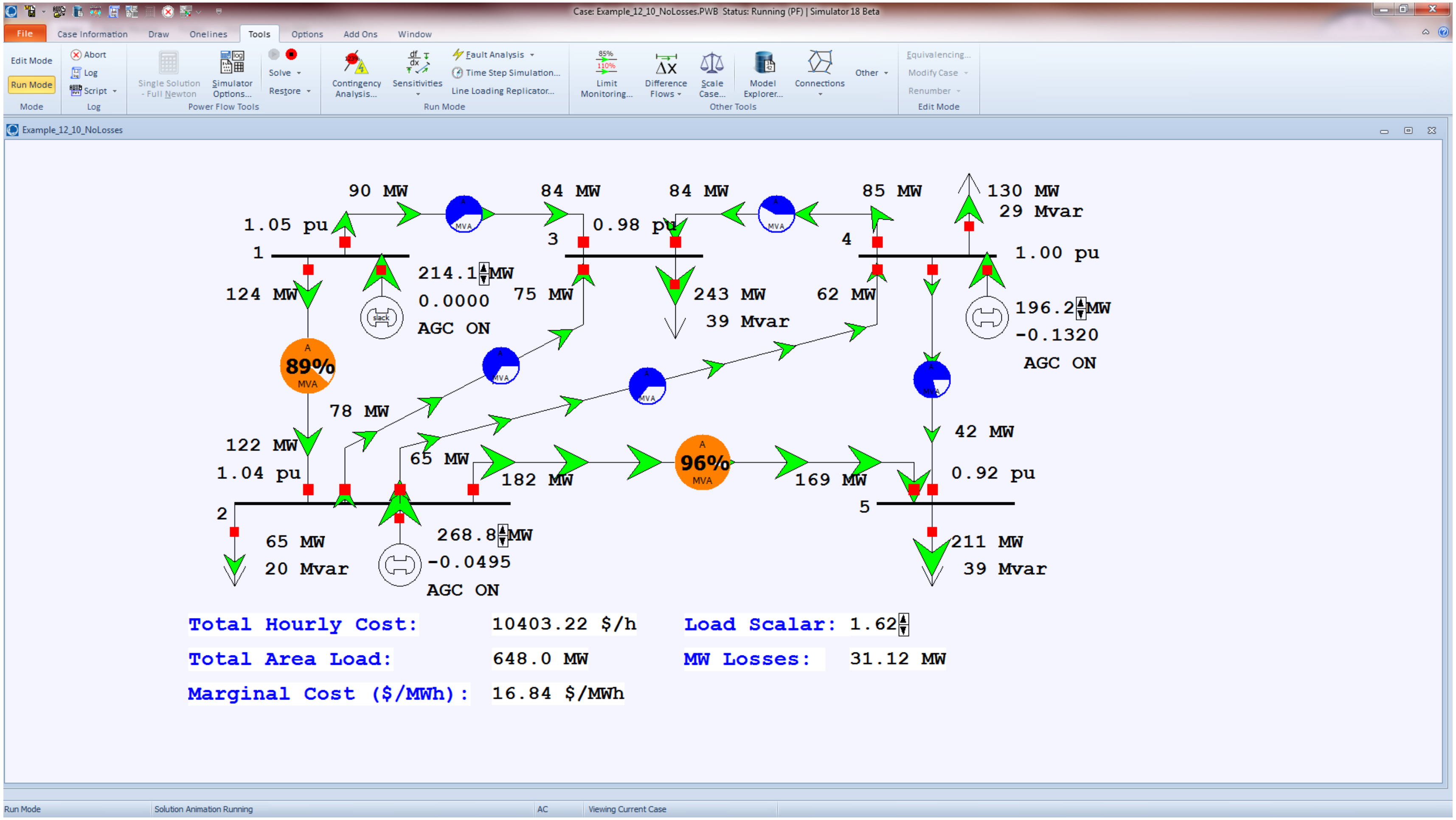
Economic Dispatch and OPF

- A key shortcoming of economic dispatch is it does NOT consider transmission system constraints
  - That is, just dispatching the system economically may lead to a situation in which we have transmission system limit violations
- OPF combines economic dispatch with the power flow to achieve an optimal dispatch
- There are several common OPF solution techniques; here we'll just consider the widely used linear programming approach

LP OPF Solution Method

- Solution iterates between
  - solving a full ac power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost
- A key input is determining the sensitivities of each constraint to each control

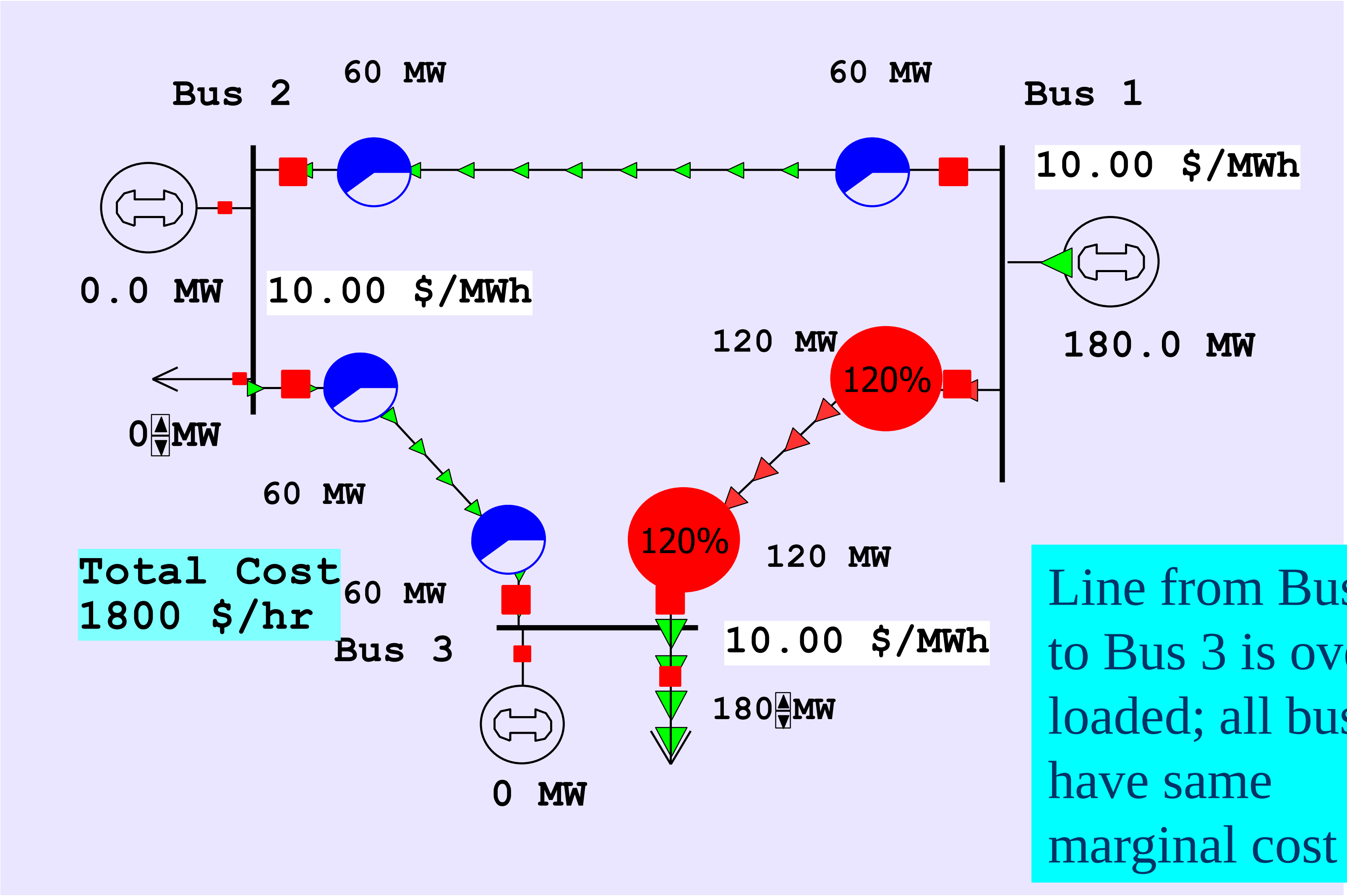
Five Bus MW Control Sensitivities



Three Bus (B3) Example

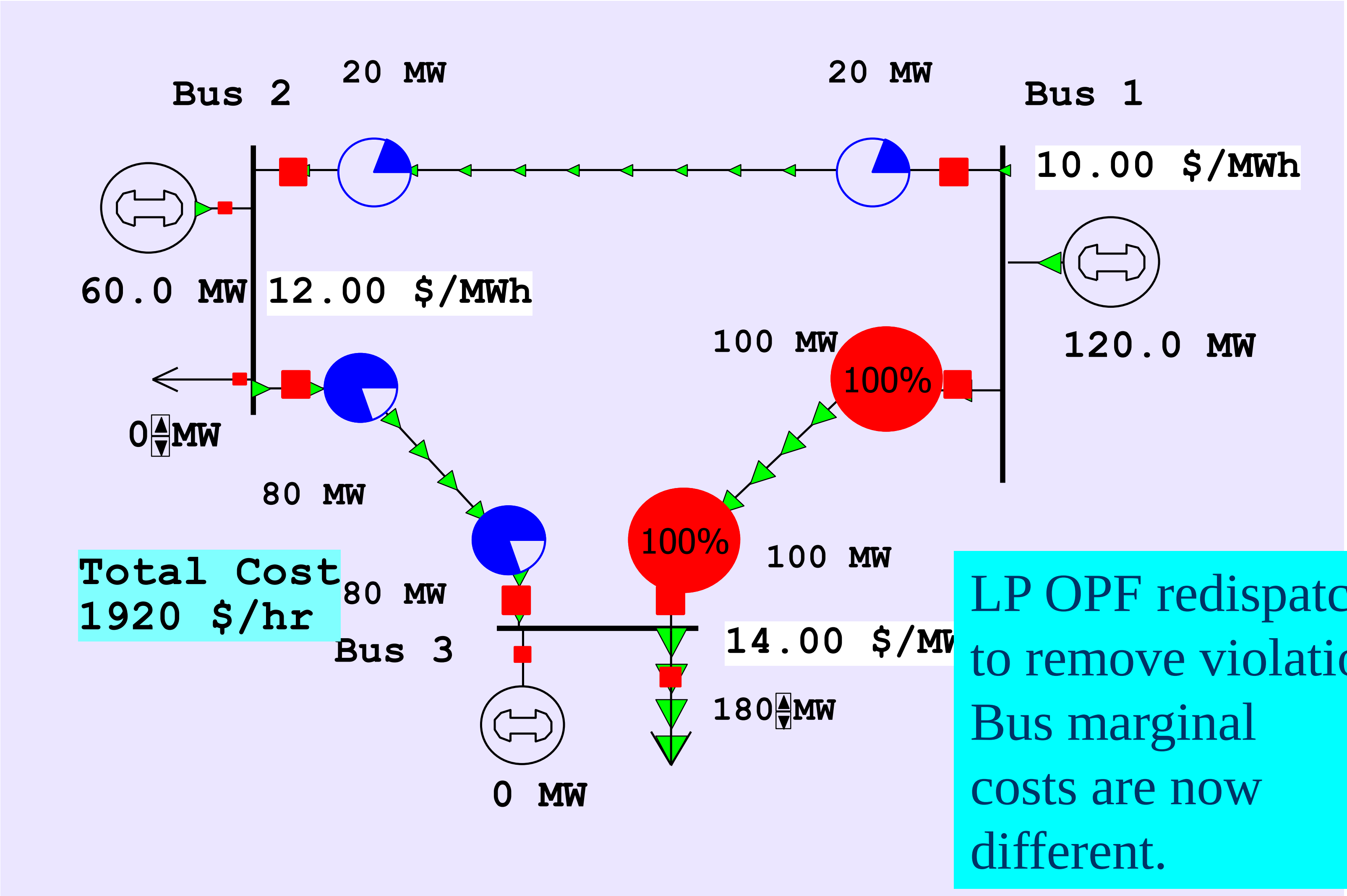
- Consider a three bus case (bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

B3 with Line Limits NOT Enforced



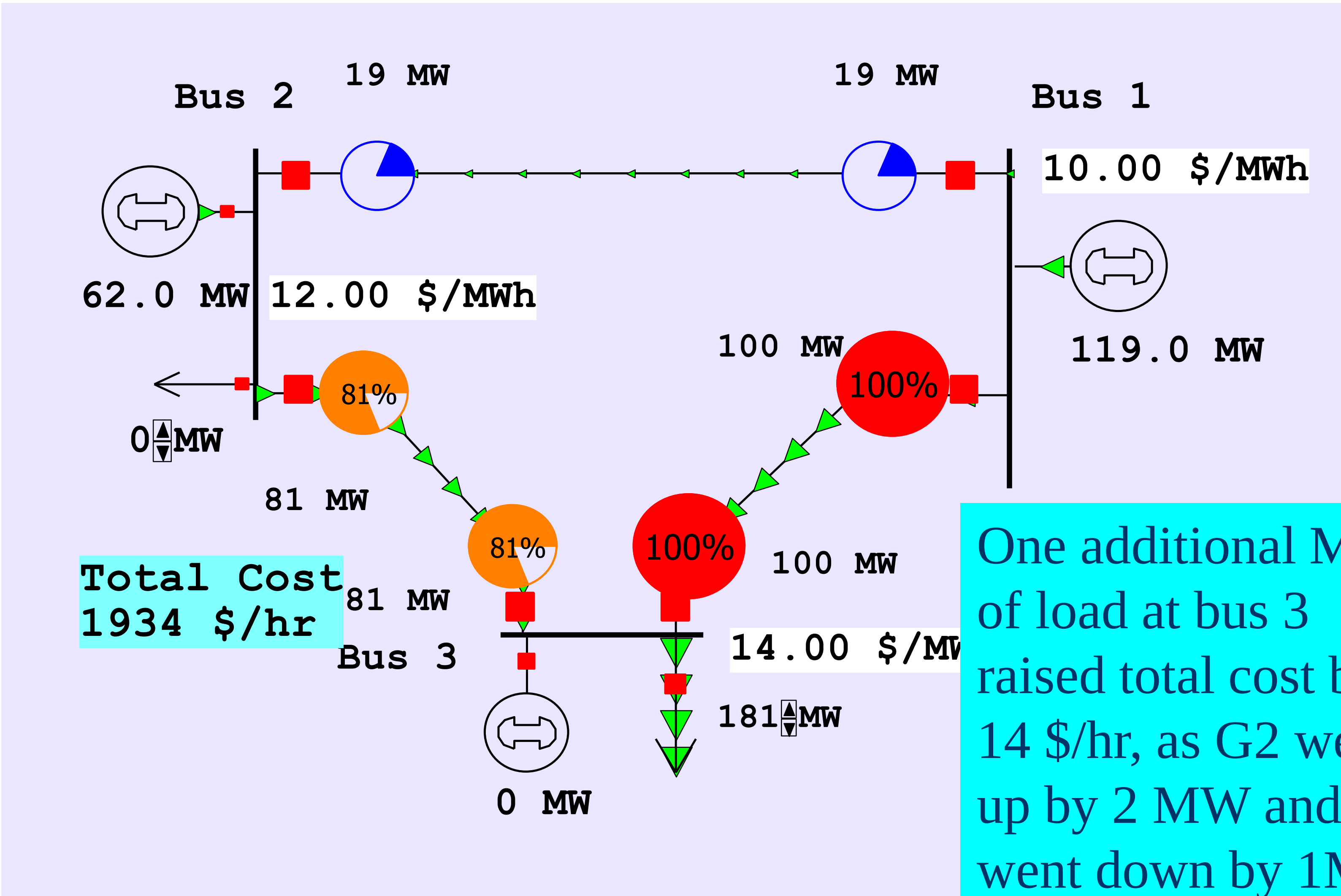
Line from Bus 1 to Bus 3 is over-loaded; all buses have same marginal cost

B3 with Line Limits Enforced



LP OPF redispatches to remove violation. Bus marginal costs are now different.

Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

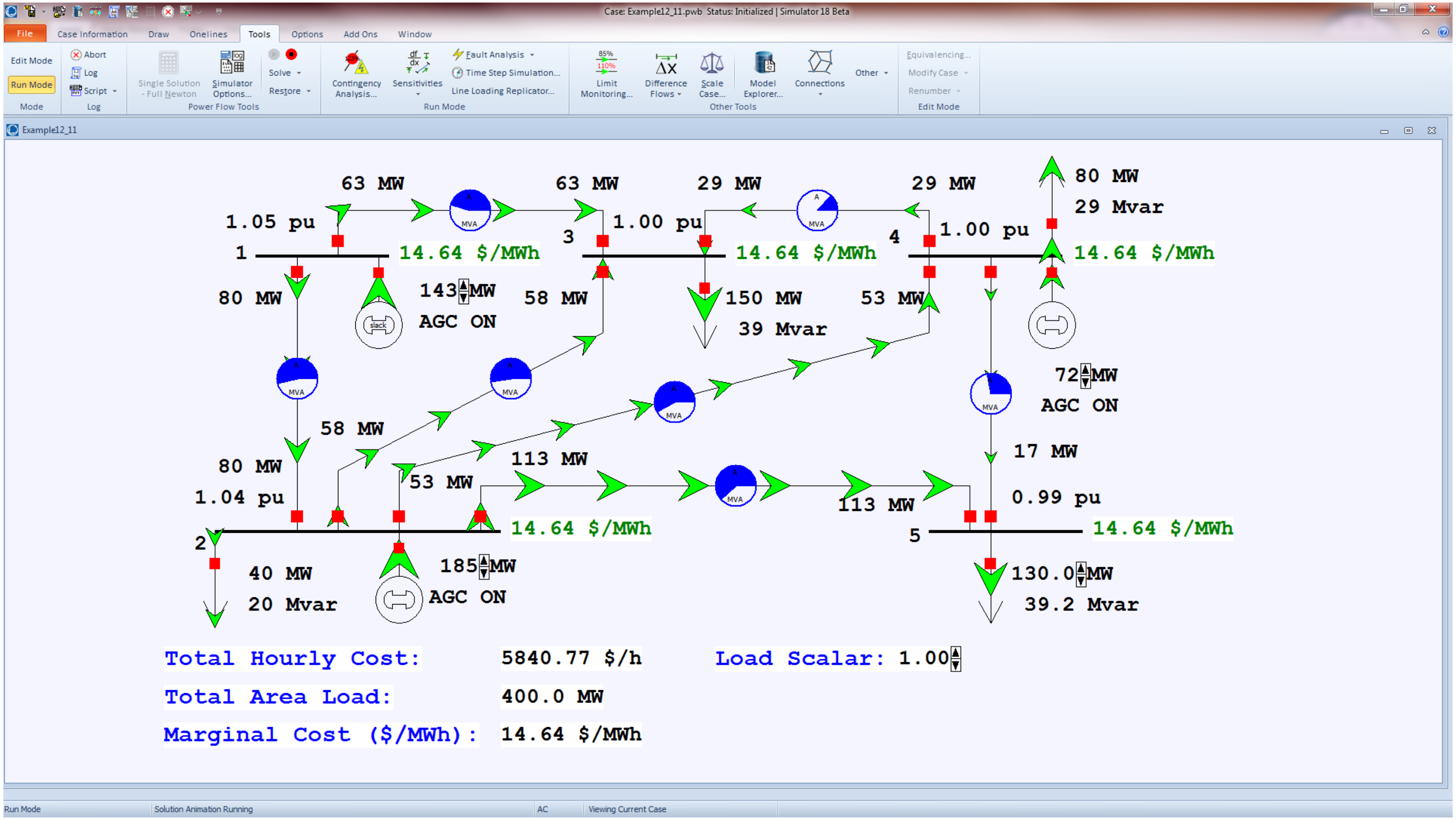
Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would "loop around" from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

Why is bus 3 LMP \$ 14 / MWh, cont'd

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - Pg1 + Pg2 = 1 MW
  - 2/3 Pg1 + 1/3 Pg2 = 0; (no more flow on 1-3)
- Solving requires we up Pg2 by 2 MW and drop Pg1 by 1 MW -- a net increase of \$14.

Five Bus OPF Solution and LMPs



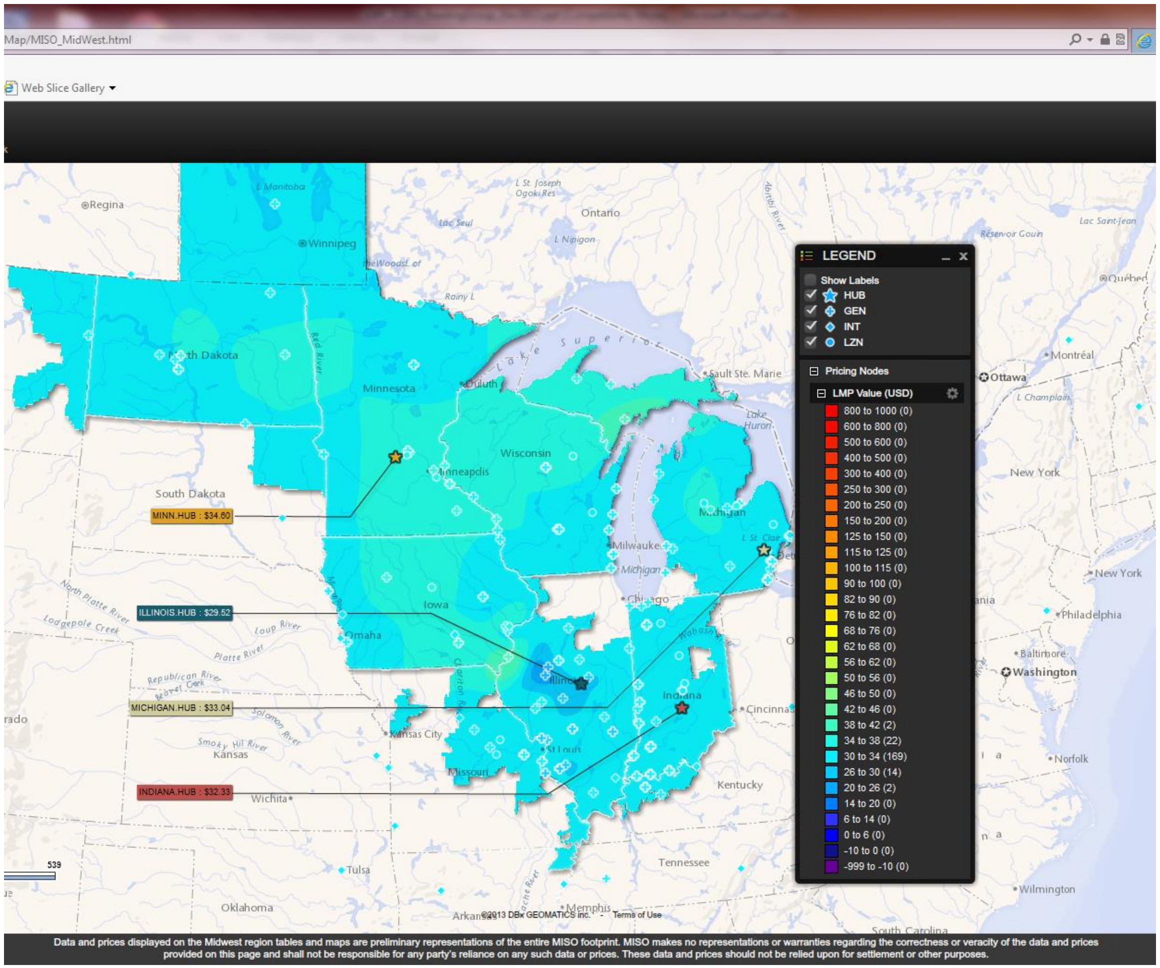
Typical Electricity Markets

- Electricity markets trade a number of different commodities, with MWh being the most important
- A typical market has two settlement periods: day ahead and real-time
  - Day Ahead: Generators (and possibly loads) submit offers for the next day; OPF is used to determine who gets dispatched based upon forecasted conditions. Results are financially binding
  - Real-time: Modifies the day ahead market based upon real-time conditions.

Payment

- Generators are not paid their offer, rather they are paid the LMP at their bus, the loads pay the LMP.
- At the residential/commercial level the LMP costs are usually not passed on directly to the end consumer. Rather, they these consumers typically pay a fixed rate.
- LMPs may differ across a system due to transmission system "congestion."

MISO LMP Contour



https://www.misoenergy.org/LMPContourMap/MISO_MidWest.html

Questions?