Introduction to Power System Analysis

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Agenda

- Power System Notation
- Power Flow Analysis
- Hands on Matpower and PowerWorld

Simple Power System

- Every power system has three major components
  - generation: source of power, ideally with a specified power, voltage, and frequency
  - load: consumes power; ideally with constant power consumption
  - transmission system: transmits power; ideally as a perfect conductor

Complications

- No ideal voltage sources exist
- Loads are seldom constant, and we need to balance supply and demand in real time
- Transmission system has resistance, inductance, capacitance and flow limitations
- Simple system has no redundancy so power system will not work if any component fails

Notation - Power

- Power: Instantaneous consumption of energy
- Power Units
- Watts = voltage x current for dc (W)
- kW – 1×10^3 Watt
- MW – 1×10^6 Watt
- GW – 1×10^9 Watt
- Installed U.S. generation capacity is about 900 GW (about 3 kW per person)
- Maximum load of Champaign/Urbana about 300 MW

Notation - Energy

- Energy: Integration of power over time; energy is what people really want (and pay for) from a power system
- Energy Units
- Joule = 1 Watt-second (J)
- kWh – Kilowatthour (3.6 x 106 J)
- Btu – 1055 J; 1 MBtu=0.292 MWh
- U.S. electric energy consumption is about 3600 billion kWh (about 13,333 kWh per person, which means on average we each use 1.5 kW of power continuously)

Review of Phasors

Goal of phasor analysis is to simplify the analysis of constant frequency ac systems

$$v(t) = V_{max} \cos(\omega t + \theta_V)$$

$$i(t) = I_{max} \cos(\omega t + \theta_I)$$

Root Mean Square (RMS) voltage of sinusoid

$$\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{V_{max}}{\sqrt{2}}$$

Complex Power

Power

Complex Power, cont'd

Average Power

Phasor Representation

Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$

Phasor notation is developed by rewriting using Euler's identity

$$v(t) = \sqrt{2}V\cos(\omega t + \theta_V)$$

$$v(t) = re[\sqrt{2}Ve^{j\omega t}e^{\theta_V}]$$

Phasor Representation, cont'd

The RMS, cosine-reference phasor is:

$$\bar{V} = Ve^{j\theta} = V\angle\theta_V$$

$$\bar{V} = V(\cos\theta_V + j\sin\theta_V)$$

$$\bar{I} = I(\cos\theta_I + j\sin\theta_I)$$

Complex Power

$$S = \bar{V}\bar{I}^*$$
$$= VIe^{j(\theta_V - \theta_I)}$$
$$= VI[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + jQ$$

P: real power (W, kW, MW)

Q: reactive power (var, kvar, Mvar)

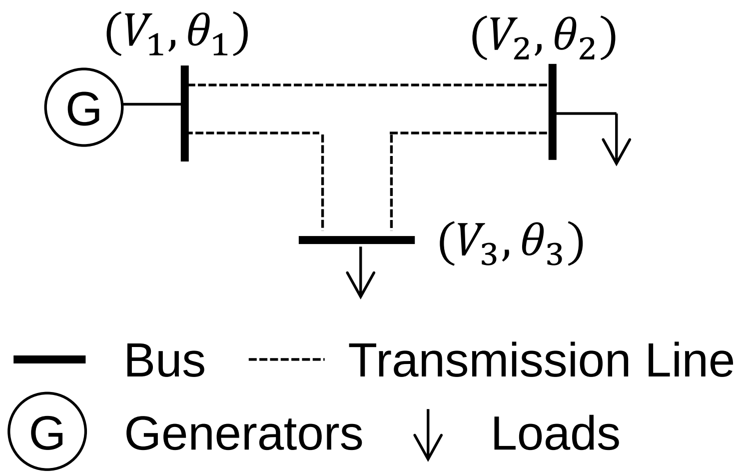
S: complex power (va, kva, Mva)

Power factor (pf): $\cos(\theta_V - \theta_I)$

If current leads voltage, then pf is leading

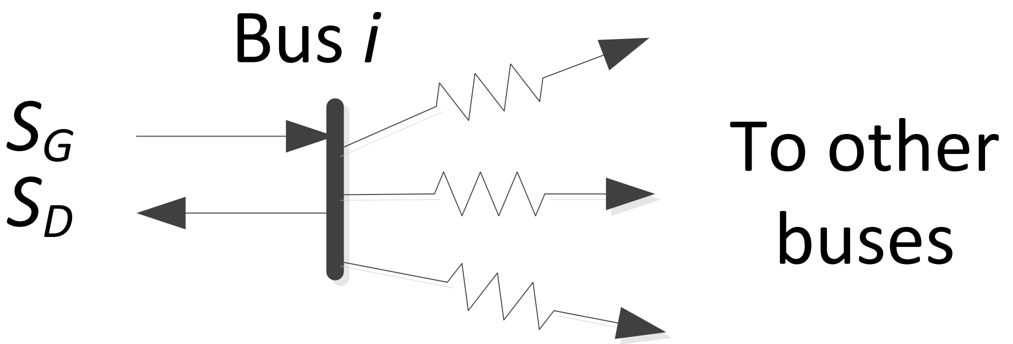
If current lags voltage then pf is lagging

Power Flow Analysis



- The *power flow analysis* is the process of solving the steady state of the power system
  - Steady state: voltage magnitude and angle for each bus
  - Generator: modeled as constant power delivery
  - Loads: modeled as constant power consumption
  - Transmission line: modeled as constant impedance

Power Flow Analysis



$$S_G - S_D = \bar{V}_i \bar{I}_i^{\,*}$$

($\bar{V}$, $\bar{I}$: the voltage and the current that injects into bus *i*)

$$\bar{I}_i = \sum_k \bar{I}_{ik}$$

$$\bar{I}_{ik} = \frac{(\bar{V}_i - \bar{V}_k)}{z_{ik}}$$

(*k* takes indices of all buses that connected to bus *i*; Z_{ik} specifies the impedance of transmission line connecting bus *i* and bus *k*)

Y-Bus (admittance matrix)

$$I_i = \sum_k \frac{(V_i - V_k)}{z_{ik}} = \sum_k (V_i - V_k) y_{ik}$$

$$= (-y_{i1}V_1) + (-y_{i2}V_2) +, \dots, [\sum_k (y_{ik}) V_i] +, \dots + (-y_{in})V_n$$

$$= \begin{bmatrix} -y_{i1} & -y_{i2} & \cdots & \sum_k y_{ik} & \cdots & -y_{in} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix}$$

Y-Bus (admittance matrix), cont'd

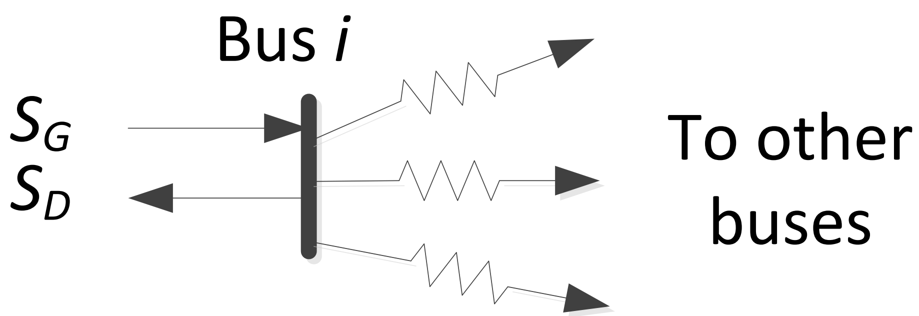
Write I_j for all buses together: $\bar{I} = Y\bar{V}$, where
($\bar{I} = [I_1, I_2, \dots, I_n]$, $\bar{V} = [V_1, V_2, \dots, V_n]$)

Construction of Y:

$$Y_{ii} = \sum_k y_{ik}$$
$$Y_{ik} = Y_{ki} = -y_{ik}$$
$$Y = G + jB$$

So we have: $I_i = \sum_k (Y_{ik} V_k)$

Power Flow Equation at Bus *j*



$$S_G - S_D = \overline{V_i}\,\overline{I_i}^{*}$$

$$S_G - S_D = \overline{V_i}\;\overline{I_i}^{*} = \overline{V_i}\sum_k \left(Y_{ik}\overline{V_k}\right)^{*}$$

$$= V_i e^{j\theta_i}\left(\sum_k (G_{ik} + jB_{ik})\, V_k\, e^{j\theta_k}\right)^{*}$$

$$= V_i e^{j\theta_i}\sum_k V_k (G_{ik} - jB_{ik}) e^{-j\theta_k}$$

$$= \sum_k V_i V_k\, (G_{ik} - jB_{ik}) e^{j(\theta_i - \theta_k)}$$

$$= \sum_k V_i V_k\, (G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k))$$

$$+j\sum_k V_i V_k\, (G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k))$$

Power Flow Equation at Bus *j*

$$P_G - P_D = \sum_k V_i V_k \left(G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right)$$

$$Q_G - Q_D = \sum_k V_i V_k \left(G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)\right)$$

- Slack bus:
  - V and θ are known, used as a reference
- PV bus, with generators connected
  - *P* and *V* are known
- PQ bus, with only load units connected
  - *P* and *Q* are known

Solving Power Flow Equations

- Assuming *m*-1 PV buses
  - Given: V_1, θ_1, $P_{G,2}$, V_2, …, $P_{G,m}$, V_m, $P_{D,m+1}$, $Q_{D,m+1}$,…, $P_{D,n}$, $Q_{D,n}$
  - Unknown: $P_{G,1}$, $Q_{G,1}$, $Q_{G,2}$, θ_2, …, $Q_{G,m}$, θ_m, V_{m+1}, θ_{m+1},…, V_n, θ_n

Newton-Raphson Methods

Assume *(m-1)* PV buses among *n* buses

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \vdots \\ \theta_n \\ V_{m+1} \\ V_{m+2} \\ \vdots \\ V_n \end{bmatrix} \qquad f(x) = \begin{bmatrix} P_2(x) - P_{G,2} + P_{D,2} \\ P_3(x) - P_{G,3} + P_{D,3} \\ \vdots \\ P_n(x) - P_{G,n} + P_{D,n} \\ Q_{m+1}(x) - Q_{G,m+1} + Q_{D,m+1} \\ Q_{m+2}(x) - Q_{G,m+2} + Q_{D,m+2} \\ \vdots \\ Q_n(x) - Q_{G,n} + Q_{D,n} \end{bmatrix}$$

Multi-Variable Example

Multi-variable Example, cont'd

N-R Power Flow Solution

The most difficult part of the algorithm is determining and inverting the n by n Jacobian matrix, $\mathbf{J}(\mathbf{x})$

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_n} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_n} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_n(\mathbf{x})}{\partial x_1} & \frac{\partial f_n(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\mathbf{x})}{\partial x_n} \end{bmatrix}$$

Other Power Flow Solution

Divide the Jacobian matrix into four sub-matrices:

$$J(x) = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

Decoupled power flow:

$$M(x) = \begin{bmatrix} \frac{\partial P}{\partial \theta} & 0 \\ 0 & \frac{\partial Q}{\partial V} \end{bmatrix}$$

Other Power Flow Solution

Fast decoupled power flow, assuming $\theta_i - \theta_k \approx 0$, $V_i \approx 1$, $G_{ik} \ll B_{ik}$

So we have: $\frac{\partial P}{\partial \theta} \approx -\tilde{B} = -\begin{bmatrix} B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \vdots \\ B_{nn} & \cdots & B_{nn} \end{bmatrix}$

$$\frac{\partial Q}{\partial V} \approx -\tilde{\tilde{B}} = \begin{bmatrix} B_{m+1,m+1} & \cdots & B_{m+1,n} \\ \vdots & \vdots & \vdots \\ B_{n,m+1} & \cdots & B_{nn} \end{bmatrix}$$

$$M(x) = \begin{bmatrix} -\tilde{B} & 0 \\ 0 & -\tilde{\tilde{B}} \end{bmatrix}$$

DC Power Flow Analysis

Assumption: small deviation flat voltage profile, $V_i \approx 1$ and $\theta_i \approx 0$

$$\frac{\partial P}{\partial \theta} \approx -\tilde{B}$$

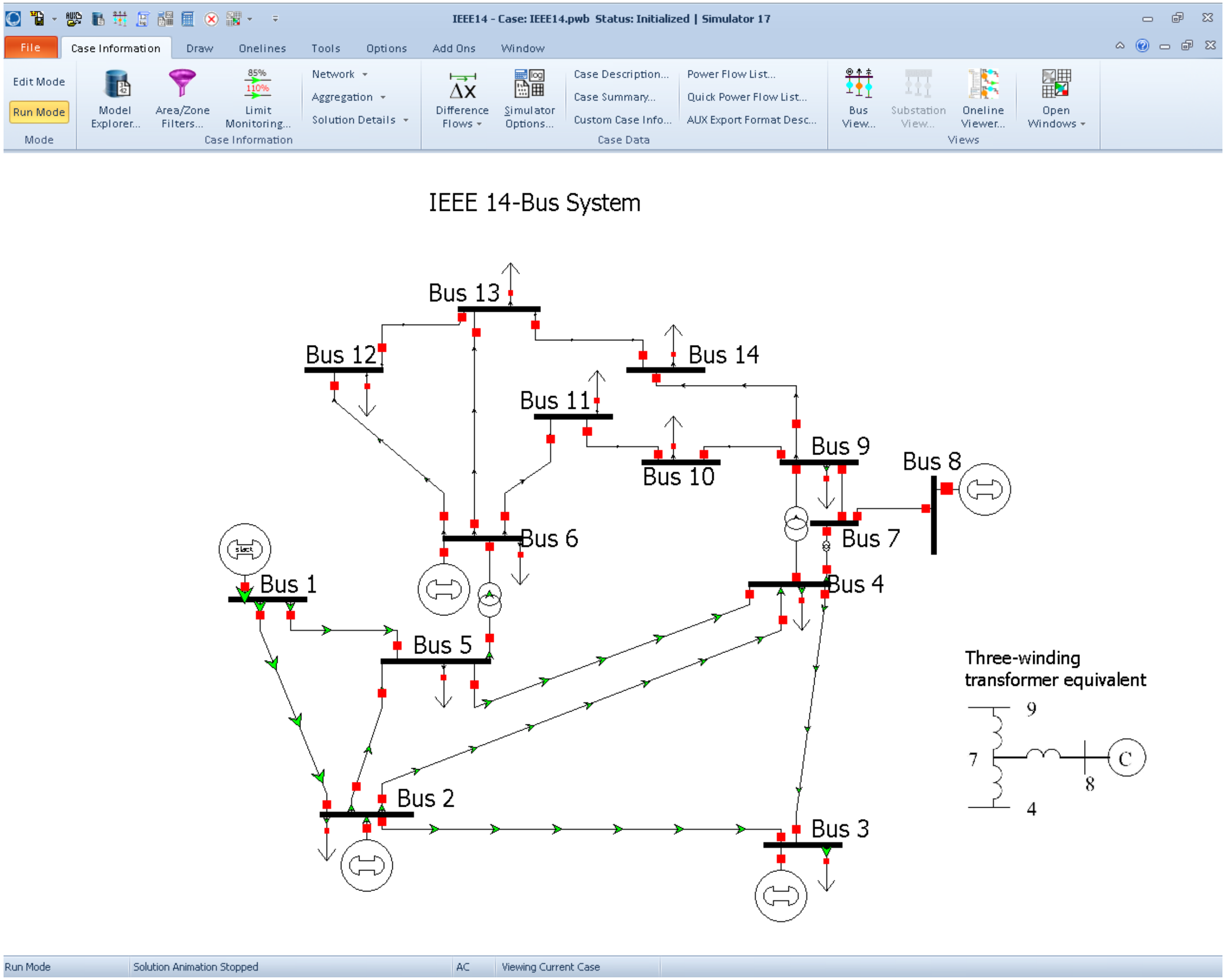
The ultimate steady state: $P = P_0 + \partial P,\ \theta = \theta_0 + \partial\theta.$ In the flat voltage profile, $P_0 = 0,\ \theta_0 = 0$

$$P \approx -\tilde{B}\theta$$

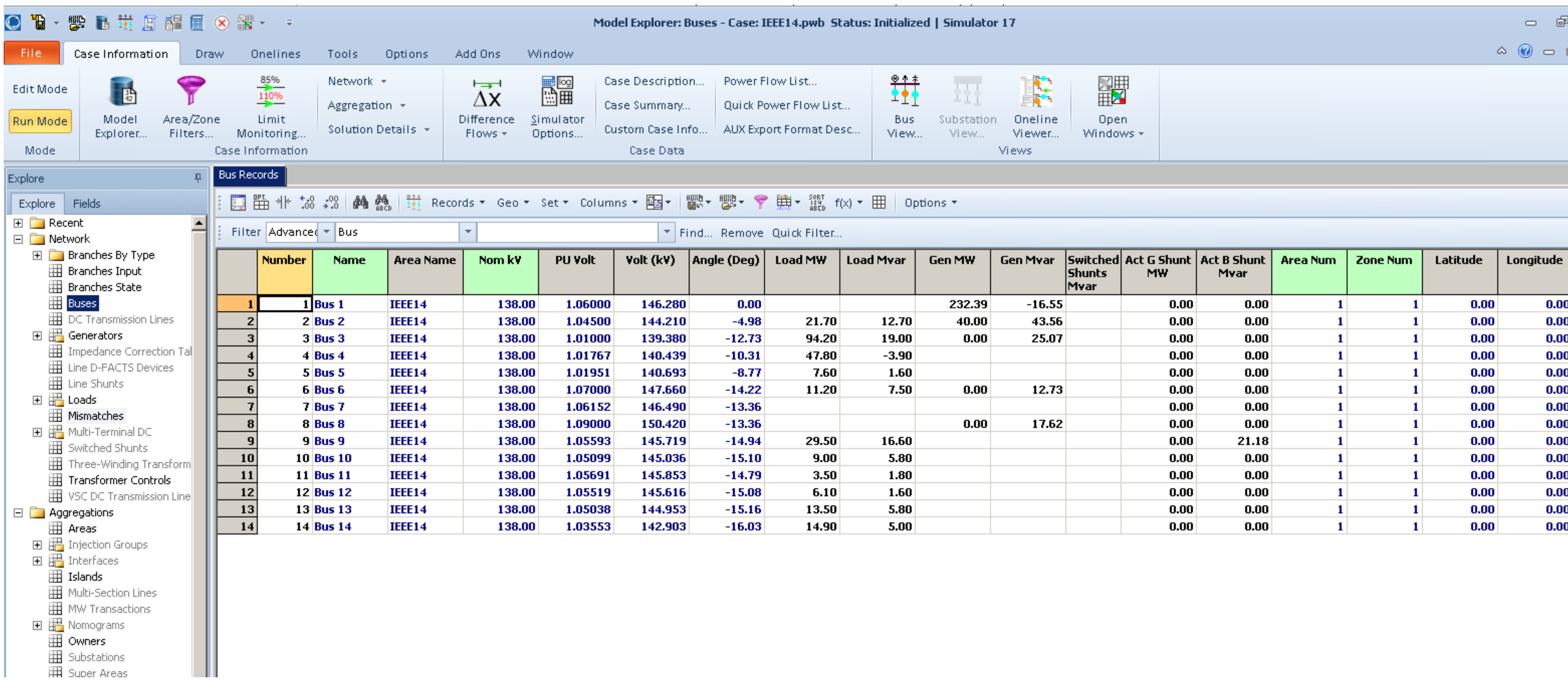
Matpower

- http://www.pserc.cornell.edu/matpower/
- **runpf**: run power flow analysis
- **runopf**: solves an optimal power flow
- **makeYbus**: Builds the bus admittance matrix and branch admittance matrices.

PowerWorld


IEEE14 - Case: IEEE14.pwb Status: Initialized | Simulator 17
File
Case Information
Draw
Onelines
Tools
Options
Add Ons
Window
Edit Mode
Run Mode
Mode
Model Explorer...
Area/Zone Filters...
Limit Monitoring...
Network
Aggregation
Solution Details
Case Information
Difference Flows
Simulator Options...
Case Description...
Case Summary...
Custom Case Info...
Power Flow List...
Quick Power Flow List...
AUX Export Format Desc...
Case Data
Bus View...
Substation View...
Oneline Viewer...
Open Windows
Views
IEEE 14-Bus System
Bus 13
Bus 12
Bus 14
Bus 11
Bus 10
Bus 9
Bus 8
Bus 6
Bus 7
Bus 1
Bus 4
Bus 5
Bus 2
Bus 3
Three-winding transformer equivalent
9
7
8
4
C
Run Mode
Solution Animation Stopped
AC
Viewing Current Case

PowerWorld, cont'd



	Number	Name	Area Name	Nom kV	PU Volt	Volt (kV)	Angle (Deg)	Load MW	Load Mvar	Gen MW	Gen Mvar	Switched Shunts Mvar	Act G Shunt MW	Act B Shunt Mvar	Area Num	Zone Num	Latitude	Longitude
1	1	Bus 1	IEEE14	138.00	1.06000	146.280	0.00			232.39	-16.55		0.00	0.00	1	1	0.00	0.00
2	2	Bus 2	IEEE14	138.00	1.04500	144.210	-4.98	21.70	12.70	40.00	43.56		0.00	0.00	1	1	0.00	0.00
3	3	Bus 3	IEEE14	138.00	1.01000	139.380	-12.73	94.20	19.00	0.00	25.07		0.00	0.00	1	1	0.00	0.00
4	4	Bus 4	IEEE14	138.00	1.01767	140.439	-10.31	47.80	-3.90				0.00	0.00	1	1	0.00	0.00
5	5	Bus 5	IEEE14	138.00	1.01951	140.693	-8.77	7.60	1.60				0.00	0.00	1	1	0.00	0.00
6	6	Bus 6	IEEE14	138.00	1.07000	147.660	-14.22	11.20	7.50	0.00	12.73		0.00	0.00	1	1	0.00	0.00
7	7	Bus 7	IEEE14	138.00	1.06152	146.490	-13.36						0.00	0.00	1	1	0.00	0.00
8	8	Bus 8	IEEE14	138.00	1.09000	150.420	-13.36			0.00	17.62		0.00	0.00	1	1	0.00	0.00
9	9	Bus 9	IEEE14	138.00	1.05593	145.719	-14.94	29.50	16.60				0.00	21.18	1	1	0.00	0.00
10	10	Bus 10	IEEE14	138.00	1.05099	145.036	-15.10	9.00	5.80				0.00	0.00	1	1	0.00	0.00
11	11	Bus 11	IEEE14	138.00	1.05691	145.853	-14.79	3.50	1.80				0.00	0.00	1	1	0.00	0.00
12	12	Bus 12	IEEE14	138.00	1.05519	145.616	-15.08	6.10	1.60				0.00	0.00	1	1	0.00	0.00
13	13	Bus 13	IEEE14	138.00	1.05038	144.953	-15.16	13.50	5.80				0.00	0.00	1	1	0.00	0.00
14	14	Bus 14	IEEE14	138.00	1.03553	142.903	-16.03	14.90	5.00				0.00	0.00	1	1	0.00	0.00

Thanks