Generating Random Topology Power Grids

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Abstract

Simulation based on standard models is often used as part of the engineering design process to test theories and exercise new concepts before actually placing them into operation. In order to tackle the problem of likely widespread catastrophic failures of electric power grids, an autonomously reconfigurable power system will have to rely on wide-area communication systems, networked sensors, and restorative strategies for monitoring and control. Standard practice is to use simulation of a small number of certain historical test systems to test the efficacy of any proposed design. We believe this practice has shortcomings when examining new communication system ideas.

In this paper we develop the means for producing power grids with scalable size and randomly generated topologies. These ensembles of networks can then be used as a statistical tool to study the scale of communication needs and the performance of the combined electric power control and communication networks. The topological and system features of the randomly generated power grids are compared with those of standard power system test models as a "sanity check" on the method.

1. Introduction

In the United States the bulk electric power system is operating ever closer to its reliability limits. During the past decade there have been numerous efforts aimed at finding preventive and/or restorative methods that would prove effective in dealing with likely widespread catastrophic failures caused either by unanticipated disturbances, such as the blackout on August 14 2003, or intentional attacks [1~9]. It is widely believed that there are inseparable interdependencies between reliable operations of electric power grids and the efficient placement and operation of related telecommunication networks. By an autonomously reconfigurable power grid we mean one that contains an appropriate level of operational strategies, either automatic or human-intervened, such that during and after unexpected natural or man-made events essential system functions are restored and/or preserved. Clearly, an autonomously reconfigurable power grid must rely on wide-area measurements and controls, networked sensors, and adaptive autonomously configurable strategies.

During the process of our research to study the control mechanism for power grids with a related communication system, we found that, due to the requirement of statistical analysis in the research problem, it is fundamental to have a large number of test power grids with appropriate topologies and scalable network size, in order to design, examine or justify any proposed implementation. For example, it is desirable to know how the communication needs (such as. bandwidth, or delay requirements, etc) grow with regard to the power grid size, given a specific control scheme, contingency sets and communication resource settings. If the growth is sub-linear, the design of the scheme will be able to take advantage of this property and choose an appropriate communication implementation accordingly.

However, due to various reasons, it is usually difficult to obtain a lot of "real" power systems for study of control & communication problems described as above. Existing standard synthetic test systems, such as IEEE test cases, cannot provide a sufficiently large group of samples neither.

Interestingly, we have noticed similar needs of scalable-size power grid test cases in the work by other researchers. For example, in [17] the author used "ring-like" power grids to study the pattern and speed of contingency or disturbance propagation. In [18] the author studied critical points and transitions in electrical power transmission network by using a "tree-like" power network model to generate power girds with scalable size. Both the models can provide some help to generate test power grids for studying our research problems. But the topologies of generated power grids from them, i.e., ring-like or tree-like structures, could not fully represent the topological characteristics of realistic power system.

We also noted that there is substantial work reported in the literature on topological characteristics of complex large networks ranging from social, natural, to man-made networks, such as human acquaintances, neural networks, the World-Wide Web, and the power grids [11,12,13,14]. There is very little work we are aware of that aimed at generating random topology power networks not only suitable for study of the topological features but also for the electrical dynamics of the system. As stated in [12] a large-scale power grid reveals some characteristics of a "small world" network. That is, despite their often-large size, there is a relatively short path between any two nodes (i.e., buses in power grids). Here the path length is measured as the number of hops along the shortest route from one end to the other. The most famous model for generating a small-world network, termed a WS model, begins with a ring lattice with N nodes in which every node is connected to its first k neighbors ($k/2$ on either side of the node). Then each edge of the lattice is randomly rewired with probability p in a way such that self-connections and duplicate edges are forbidden. In order to have a sparse but connected network it is required that

$$N >> k >> \ln(N) >> 1 \qquad (1)$$

Obviously the average nodal degree of the resulting topology is still k. Empirical results of power grids indicate that the average nodal degree $<k>$ is about 2.67 for WSCC system and 4.78 for NYISO system, which is quite small. Note that (1) requires N be far larger than 2.67 (or 4.78) while at the same time far less than $e^{2.67} \approx 14$ (or $e^{4.78} \approx 119$). Therefore, it is very hard if not impossible to create a connected power grid topology by using a WS-small world model. Additionally, the random network models discussed in [12] are concerned with only the topological features of the network with little attention given to specific system features, such as bus locations, transmission line length and impedance, generation/load settings etc. Therefore, in this paper we propose a new model for generating sparse and connected topologies associated with realistic power grids while preserving their "small world" characteristics.

In this model the first generating step of a grid is to form a random topology that preserves simpleness[1] and connectedness. That is, the nodal locations, which represent buses in a power grid, are specified according a probalisitc distribution function. The branches, which represent the transmission lines, are selected according to distance criterion and a corresponding random distribution function. In the second step, transmission line impedances, generation and load set-points, and generator parameters are all determined randomly for further eigenvalue analysis. The advantage of this proposed model is that it can produce a large number of test power grids with appropriate topology and scalable size, which could be used to study statistical characteristics of system performance. In addition to the design of control and communication networks for power grids, this model can also be applied to study other research problems such as evaluation of communication protocols for power grids or other theoretical study of power grids, which is related with scalable network size.

In section II we present the model for generating random-topology power grids. In section III and IV we characterize the topological features and electrical system features of simulated power grids. A comparison is provided between power grids generated with our technique and conventional power grid simulation models. Section V introduced Maximum Likelihood experiments to vilify the effectiveness of the proposed model and section VI concludes the paper.

2. Random-topology power grids

The proposed model was created in order to form randomly generated but realistic power grids. Currently we model only high-voltage transmission networks. The system topology and electrical settings are created with specific random distribution functions and the generated topology preserves both simpleness and connectedness. Simpleness means that there are no self-loops and connectedness means there are no islands. These two requirements are consistent with real-world interconnected power networks.

A. Formation of Random Topologies

The process contains three steps.

(s1) Inside a fixed area, given the expected number $\widetilde{N}$ of nodal locations, N nodal locations are selected according to a random distribution function. For example, either Uniform or Poisson distribution works well in this case (Corresponding model termed Uniform-RT or Poisson-RT respectively). Note that in the resulting graph with Poisson distribution applied, it is possible that $N \neq \widetilde{N}$. Uniform distribution is applied if the buses are considered to be more likely evenly spread in the area. Poisson distribution is adopted for an unevenly expected distribution.

(s2) Given the requirement of distance limitation

[1] An undirected graph is called "simple" if the graph has no self-loops and no duplicate links between a same pair of nodes.

$d_{\min} \le d \le d_{\max}$ and the expected link length distribution $p(d)$, the neighboring links of each bus are selected. Observation of synthetic model or empirical power grids tells that the line impedances of a grid approximate either a Gamma distribution or an Exponential distribution (with specific parameter settings of course). In our model it is assumed that line impedance is proportional to its physical length. Therefore the expected link length distribution function is set as either a Gamma or an Exponential distribution. Then a random number k of transmission lines are picked at random from each node's neighboring links according to a specified distribution function $p(k)$ with expected value equal to $<k>$ (set as 2.67 in our simulation). The random distribution of select transmission lines can either be Exponential or Poisson. The simulation shows that Poisson distribution resulting connected topologies with a much higher probability than Exponential distribution.

(s3) The former two steps have guaranteed a simple topology. Further inspection is performed to check if the resulting topology is connected. If connected, stop; otherwise, repeat from step (s1). Finally a connected topology will be resulted with N nodes (representing buses in the grid) and m links (representing transmission lines in the grid), as shown in Fig.1 for an example, which is described by network connectivity matrix A defined as a m by N binary:

$$A(i_l, i_n) = \begin{cases} 1, & \text{if } i_l^{\text{th}} \text{ link starts from } i_n^{\text{th}} \text{ node;} \\ -1, & \text{if } i_l^{\text{th}} \text{ link ends from } i_n^{\text{th}} \text{ node;} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

B. Assignment of Power Grid Parameters

The impedance of each transmission line is assumed to be proportional to its physical length plus a small random deviation. That is,

$$Z_{pr} = Z_0 L_{line} + \varepsilon_z \tag{3}$$

where $Z_0 = r + jx$ is the line impedance per unit length, is a uniform random variable in the range of $[-\varepsilon_{z0}, +\varepsilon_{z0}]$. With the settings of power base $S_B = 100$ (MVA) and appropriate voltage ratio V_B (kV), the line impedance is translated into per unit value, that is, $Z_{pr}(p.u.) = Z_{pr} S_B / V_B^2$. Then the network admittance can be formed as:

$$Y_{bus} = A^T \cdot diag\{Y_{pr}\} \cdot A + diag\{\varepsilon_{Yd}\} \tag{4}$$

where $Y_{pr} = 1/Z_{pr}$ is the line admittance, A is the connectivity matrix, and ε_{Yd} represents a random grounding admittance, taken as uniform variables in the range of $[0, \varepsilon_{Yd\max}]$.

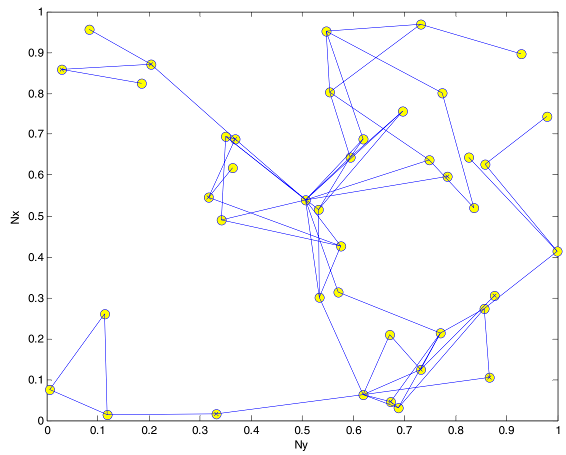


Fig.1. A connected random topology generated from the proposed Random-Topology formation model: initial nodal locations take Poisson distribution in a unit-size area, $d_{\min}$= 0, $d_{\max}$= 0.3 p.u., the selection of links take Poisson distribution with $\lambda = 2.67$.

Loads with random settings $P_l + jQ_l$ are applied on all the buses with P_l and Q_l uniformly selected from the ranges of $[P_{l\min}, P_{l\max}]$ and $[Q_{l\min}, Q_{l\max}]$ respectively. Later in further eigenvalue analysis, the loads will be translated into constant passive grounding admittance as $Y_l = diag\{P_l - jQ_l / U_l^2\}$ and integrated into the network admittance matrix, that is, $Y = Y_{bus} + Y_l$.

With probability p_{gen} generator buses are randomly picked from all of the N buses, the first one is chosen as the slack-bus which will later be assigned a very large inertia. Then the generator inertia M and transient impedance jX_d can be set from random distribution functions, in our simulations, M as a uniform variable from the range of $[M_{\min}, M_{\max}]$, and X_d from the range of $[X_{d\min}, X_{d\max}]$. The internal generator voltage potential E is selected uniform randomly from the range of $[E_{\min}, E_{\max}]$ and the internal generator rotor angle δ is set as Gaussian random variable with mean m_δ and deviation σ_δ. Without loss of generality, set m_δ equal to zero. We use the settings of σ_δ to indicate different operation conditions, namely in our simulation, $\sigma_\delta = 5^\circ$ is for the steady-state operation, and $\sigma_\delta = 30^\circ, 60^\circ, or\ 90^\circ$ represents the small, medium, or large-disturbance condition respectively.

3. Topological features

The metrics adopted to characterize topological features of a power grid include: the network size (i.e, number of nodes N, and number of links m), the average path length $<l>$, the average nodal degrees $<k>$, Pearson degree correlation coefficient ρ, and the fraction of the nodal degrees which is larger than the average degree of a node seen at the end of a randomly selected link, i.e. $r(k_i > \bar{k})$. Pearson coefficient is chosen because Newman [15] observed that for some kinds of networks, it is consistently positive while for other kinds it is negative; therefore it might be used as a tool for graph assortment. However, the work [13] of Whitney and Alderson found that in many cases of practical interest, an observed value of ρ may be explained simply by the constraints imposed by its deviance, and empirically such constraints are often the case for observed $\rho < 0$. In other cases, most often for $\rho > 0$, the possible values of ρ expand into a much larger range and other explanations must be sought. In our simulations we wish to verify if Pearson degree correlation coefficient is an appropriate metric to classify power grid topology. For a topology of N nodes and m links, suppose the nodal degree sequence is $\{k_1, k_2, \cdots, k_N\}$ which has been ordered in ascending sequence as $k_1 \le k_2 \le \cdots \le k_N$; l_{ij} is counted in number of hops along the shortest path between any pair of nodes. The definition of the metrics or terms mentioned above are listed as follows:

$$< l >= \frac{2}{N(N-1)} \sum_{(i,j)} l_{ij} \tag{5}$$

$$< k >= \frac{1}{N} \sum_i K_i \tag{6}$$

$$\bar{k} = (2m)^{-1} \sum_{(i,j)} (k_i + k_j) = (2m)^{-1} \sum_i (k_i)^2 \tag{7}$$

$$r(k_i > \bar{k}) = \frac{\left\| \{k_i : k_i > \bar{k}\} \right\|}{N} \tag{8}$$

$$\rho = \frac{\sum_{(i,j)} (k_i - \bar{k})(k_j - \bar{k})}{\sqrt{\sum_{(i,j)} (k_i - \bar{k})^2 \sum_{(i,j)} (k_j - \bar{k})^2}} \tag{9}$$

Table I presents the topological characteristics of several empirical power networks such as IEEE standard test model system, WSCC and NYISO power grids. Further examination of Table I shows that the average nodal degree $<k>$ and the fraction of $r(k_i > \bar{k})$ exhibit quite stable values regardless of network size. While the average path length $<l>$ grows proportionally with $\ln(N)/\ln(< k >)$, consistent with the results indicated in [11]. The Pearson correlation coefficients may take positive or negative values, but very close to zero. As stated in [13], the possible range of Pearson coefficient ρ of WSCC-4941 bus system, with random rewiring applied in a way so as to keep its degree sequence unchanged, is quite wide, as $\rho_{\min}$= -0.69 and $\rho_{\max}$= 0.90. Table II displays the topological characteristics of random-topology power grids, averaged over 100 samples for each case, with same or similar network size as those in Table I, generated from proposed Random-Topology model and WS-small world model.

TABLE I
TOPOLOGICAL CHARACTERISTICS OF EMPIRICAL POWER NETWORKS

	(N,m)	$<l>$	$<k>$	ρ	$r(k_i > \bar{k})$
IEEE-30	(30, 41)	3.31	2.73	-0.0868	0.2333
IEEE-57	(57, 80)	4.95	2.80	0.1895	0.2281
IEEE-118	(118, 186)	6.31	3.15	-0.0518	0.1949
IEEE-300	(300, 411)	9.94	2.74	-0.2137	0.2467
NY-2935	(2935,13136)	16.43	4.78	0.4593	0.1428
WSCC	(4941, 6954)	18.70	2.67	0.0035	0.2022

TABLE II
TOPOLOGICAL CHARACTERISTICS OF SIMULATED POWER NETWORKS FROM THE PROPOSED RANDOM-TOPOLOGY MODELS AND WS-SMALL WORLD MODEL

	(N,m)	$<l>$	$<k>$	ρ	$r(k_i > \bar{k})$
Poisson-30	(30, 40.3)	3.60	2.69	-0.1013	0.2883
Uniform-30	(30, 40.7)	3.52	2.71	-0.0953	0.2967
Poisson-57	(57,76.5)	4.44	2.69	-0.0498	0.2596
Uniform-57	(57, 75.8)	4.49	2.66	-0.0630	0.2798
Poisson-118	(118, 195.6)	4.22	3.32	-0.0174	0.3389
Uniform-118	(118, 194.8)	4.25	3.30	-0.0220	0.3368
Poisson-300	(300, 457.1)	5.56	3.05	-0.0012	0.3273
Uniform-300	(300, 456.4)	5.59	3.04	-0.0091	0.3252
WSsw-300*	(300, 423.0)	24.42	2.82	0.0703	0.1978

* WSsw-300 denotes the 300-bus power grids generated by Watts-Strogatz small-world model.

Comparison on the topological metrics of power grids with similar network size in Table I and Table II shows that: (a) the random-topology power grids generated by the proposed Poisson-RT or Uniform-RT model approximate very well the IEEE standard systems and WSCC power grid when the metrics as follows considered: total number of links m, average path length $<l>$, average nodal degree $<k>$, and the fraction of $r(k_i > \bar{k})$; (b) The Pearson coefficients ρ of random-topology power grids exhibit very close-to-zero negative values most of the time; and (c) While the WS-"small world" model generates the topologies with much larger average path length than that of IEEE standard test systems with comparable network size. Another prominent disadvantage of WS-"small world" model is that the probability is extremely low for this model to generate a connected power grid topology with an expected small average nodal degree.

4. Electrical system features

The eigenvalues of the power grids dynamics and in particular, the percentage and scale of those, falling in the left half plane, that produces unstable modes are important signatures of system dynamics. In [10] Tatikonda and Mitter provide a lower bound on the communication rate R (in bits/second) required for the asymptotic observability and stability of a linear discrete time-varing system. This rate is given by $R \geq \sum_{\lambda(C)} \max\{0, \log|\lambda(C)|\}$,where C is the characteristic matrix of the system, assumed to be linear, and $\lambda(C)$ are the eigenvalues of C. Obviously, in above formula, only the eigenvalues with magnitude greater than 1, i.e., corresponding to unstable mode, contribute to the require rate. Generally speaking, power grids dynamics is a nonlinear system. Equivalent linearization of the system equation can be applied at each running state to obtain its time-varying characteristic matrix and make eigen-analysis possible for the resulting equivalent time-varying linear system.

In this paper we choose to use the statistical distribution of the eigenvalues to characterize the system features of the generated power grids by the proposed random-topology model. Without consideration of damping effects, the inertial model of the synchronous machines can be described as:

$$\begin{cases} \dfrac{d\delta_i}{dt} = \omega_i \\ \dfrac{M_i}{\omega_R}\dfrac{d\omega_i}{dt} = P_{mi} - P_{ei} \end{cases} ,i = 1,2,\cdots,n_g \qquad (10)$$

The electrical power output of machine i can be expressed as:

$$P_{ei} = E_i^2 G_{ii} + \sum_{j\neq i} E_i E_j (B_{ij} \sin\delta_{ij} + G_{ij}\cos\delta_{ij}) \quad (11)$$

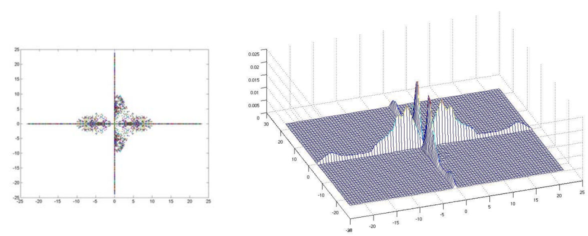
where $\delta_{ij} = \delta_i - \delta_j$, E_i is the constant internal voltage potential of machine i, $Y_{ii} = G_{ii} + jB_{ii}$ and $Y_{ij} = G_{ij} + jB_{ij}$ are respectively the diagonal element and the off-diagonal elements of reduced-network admittance matrix $\overline{Y}$ obtained from network admittance matrix Y after the application of Kron reduction so that the resulting equivalent network only contains generator buses. With linearization of P_{ei} at a set of given generator angle variables denoted by $\delta_0 = [\delta_{10}\ \delta_{20} \cdots \delta_{n_g 0}]^T$, we get the set of linearized differential equations for the system:

$$\frac{M}{\omega_R}\frac{d^2\delta_{i\Delta}}{dt^2} + \sum_{j\neq i} E_i E_j (B_{ij}\cos\delta_{ij0} - G_{ij}\sin\delta_{ij0})\delta_{ij\Delta} = 0$$
$$i = 1,2,\cdots,n_g \qquad (12)$$

where $\delta_{i\Delta}$ is the increment from the initial given value δ_{i0} of the generator angle for machine i. From (12) the characteristic matrix can be derived and corresponding eigenvalues can be calculated at given δ_0 .

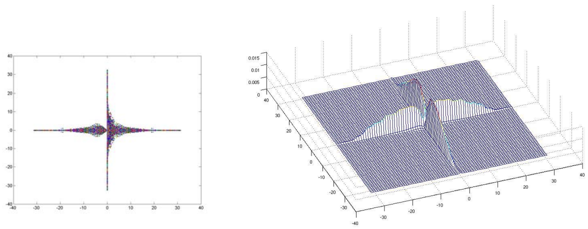
In order to take into account different contingency disturbances, a set of Gaussian random variables with $(m_\delta, \sigma_\delta)$ are given to δ_0 . As mentioned in Section II.B, the settings of $m_\delta = 0, \sigma_\delta = 90^\circ$ represents large angel disturbance occurs in the system. Consequently system eigenvalues of (12) can be obtained for a large number of disturbance conditions to form a statistical distribution.

The eigenvalues distribution of IEEE-30, 118, 300 system, over several thousand cases of large disturbances, are respectively displayed in Fig. 2, Fig. 3 and Fig. 4. In each figure, (a) exhibits the distribution range of all the eigenvalues in the complex plane: x-axis is for real part, y-axis is imaginary part; (b) presents the distribution histogram of the collection of eigenvalues. It can be seen that the eigenvalue distributions of these systems are very similar to each other and demonstrate strongly characterized modality.



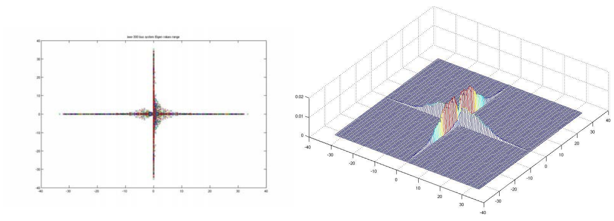
(a) (b)

Fig. 2. Distribution of eigenvalues of IEEE-30 system :
(a) the distribution range in the complex plane; (b) the distributed histogram



(a) (b)

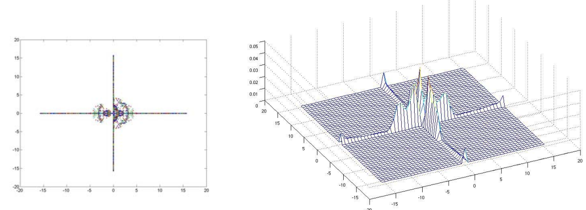
Fig. 3. Distribution of eigenvalues of IEEE-118 system :
(a) the distribution range in the complex plane; (b) the distributed histogram



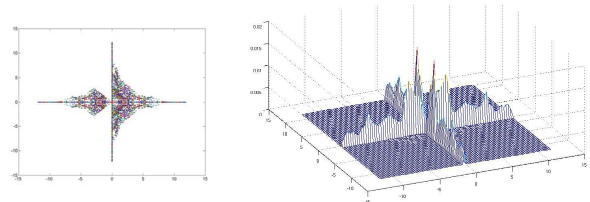
(a) (b)

Fig. 4. Distribution of eigenvalues of IEEE-300 system :
(a) the distribution range in the complex plane; (b) the distributed histogram

The eigenvalues distribution of random-topology power grids generated by our proposed RT model are shown in Fig. 5 through Fig.7. In each figure, (a/b-Poisson) denotes that the power grid is created by Poisson-RT model, (a/b-Uniform) denotes the power grid is created by Uniform-RT model. Comparison between Fig. 2~4 and Fig 5~7 demonstrate that for IEEE 30-bus system or 118-bus system, Poisson-RT model generates the power grids with better approximate of eigenvalue distributions than Uniform-RT model. While for IEEE 300-bus system, both Poisson-RT and Uniform-RT create the random-topology power grids with very close approximates of eigenvalue distribution. One possible reason to explain this is that IEEE 30-bus or 118-bus has a topology with bus locations very close to a Poisson distribution; while the bus locations in the topology of IEEE 300-bus system get closer in the direction of Uniform distribution.

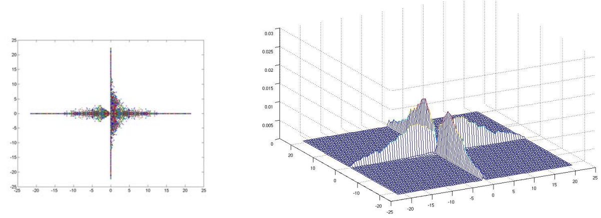


(a-Poisson) (b-Poisson)

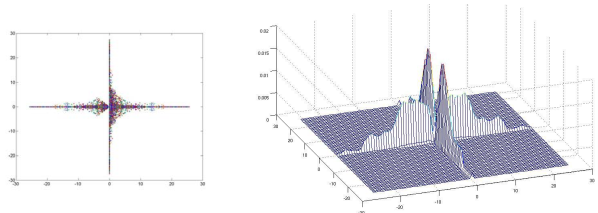


(a-Uniform) (b-Uniform)

Fig. 5. Distribution of eigenvalues of Poisson-RT 35-bus system and Uniform-RT 30-bus system :(a) the distribution range in the complex plane; (b) the distributed histogram



(a-Poisson) (b-Poisson)



(a-Uniform) (b-Uniform)

Fig. 6. Distribution of eigenvalues of Poisson-RT 103-bus system and Uniform-RT 118-bus system :(a) the distribution range in the complex plane; (b) the distributed histogram

5. Maximum likelihood

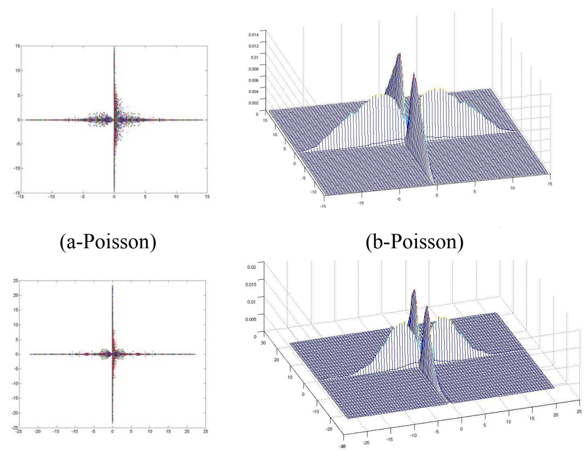
In order to objectify the selection of random topology models for power grids, we choose to use Maximum Likelihood to evaluate the validity of a model. That is, given a specific power grid topology (G) which in fact can be fully expressed by its connectivity matrix A(G), the Maximum Likelihood function of a random model (R) is defined as the probability of G assigned by R, denoted as p(A(G)|R).

In other words, the ML tells that by using R as a topology-generation tool, by how much probability, we can get a topology same as G. Please note that if the random model (R) has parameter(s) α , then ML is defined as the largest ML given the best parameter settings of R, i.e., $p^*\left(\mathrm{p(A(G)\,|\,R(}\alpha^*\mathrm{))}\right)$.

In our experiments the nodal degree sequence K instead o f A is chosen to represent a system topology. As stated in section II, the nodal degree is defined as an ascending vector of node degrees in the resulting topology termed as:

$$K = [k_1\ k_2\ \cdots k_N]. \tag{13}$$

Nodal degree sequence K is selected because first, K is much easier to compute and manipulate than the topology connectivity matrix A, and second, all the topological metrics used /defined in section II is either a function of K (such as m, <k>, ρ, $r(k_i > \bar{k})$) or closely related with it (such as the average path length <l>).



(a-Poisson) (b-Poisson)

(a-Uniform) (b-Uniform)

Fig. 7. Distribution of eigenvalues of Poisson-RT 323-bus system and Uniform-RT 300-bus system :(a) the distribution range in the complex plane; (b) the distributed histogram

Besides, K is in fact a function of A. Because an equivalent of A is system's association matrix $\widetilde{A}$, expressed as below, and each item in K equals to the sums of each row in $\widetilde{A}$. And $\widetilde{A}$ directly shapes system's admittance matrix Y_{bus} thence is closely related with the characteristic matrix of the system.

$$\widetilde{A}(i,j) = \begin{cases} 1, & \text{if the link } (i\text{-}j) \text{ exists in the graph;} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

For the model of Poisson-RT or Uniform-RT proposed in this work, it is difficult to get an analytical expression of the ML for a given topology (G). Therefore we run Monte Carlo experiments based on a specific parameter setting, termed as α_0 , to estimate the statistical distribution of nodal degree sequences K in the resulting topologies,

$$p(K) = \left(p(k_1), p(k_2), \cdots, p(k_N)\right) \tag{15}$$

Then for a given specific power grid topology (G), such as IEEE 30-bus system, supposing its nodal degree sequence is K_G , the corresponding likelihood from the RT model based on parameter settings of α_0 can be written as

$$p\left(K_G \mid RT(\alpha_0)\right) = \prod_i p(k_{Gi}) \tag{16}$$

Since here we only choose an empirically selected parameter setting, the ML based on optimistic parameter setting should be at least that as expressed in (16), that is,

$$p^*\left(K_G \mid RT(\alpha^*)\right) \ge \prod_i p(k_{Gi}) \tag{17}$$

We take Erdos-Renyi (ER) random graph model as comparison to the proposed model. For ER model, to generate a topology with a specific nodal degree sequence $K = [k_1\ k_2\ \cdots k_N]$, with N nodes and m links, where N is the size of K and m equals to $\frac{1}{2}\sum_i k_i$, the corresponding probability is

$$\Pr\left(K \mid ER(q)\right) = q^m (1-q)^{N(N-1)-m} \cdot \Pr(K \mid m) \tag{18}$$

where q is the probability of link selection in ER model, and $\Pr(K \mid m)$ is the probability to form a degree sequence as K by selecting the starting and ending nodes for m links in a N-node system, which is independent of q . It is obvious that the best parameter setting of ER is $q^* = \dfrac{m}{N(N-1)}$. For a nodal degree sequence set as $k_N \le \dfrac{N}{2}$, which is true for power grids topologies, it is known that

$$\Pr(K \mid m) \le \frac{\prod_{i=1}^{N-1} \binom{N-i}{k_{G\,N-i+1}}}{N! \binom{N(N-1)}{m}} \tag{19}$$

with the specification of $\binom{n}{k} = 1, \quad if\ n < k$.

Therefore for a specific power grid topology (G), with its nodal degree sequence K_G the corresponding ML from the ER model would be

$$p^*\left(K_G \mid ER(q^*)\right) = q^m (1-q)^{N(N-1)-m} \cdot \Pr(K_G \mid m)$$
$$\le q^m (1-q)^{N(N-1)-m} \frac{\prod_{i=1}^{N-1} \binom{N-i}{k_{G\,N-i+1}}}{N! \binom{N(N-1)}{m}} \quad (20)$$

In this work we run the experiments on IEEE 30, 57, 118, 300-bus systems based on the random models of Poisson-RT, Uniform-RT, and ER. The results are shown in Table III. For clearer expression, $-\log_{10}(\Pr)$ is shown as score instead of (Pr) in the table. Therefore, the smaller the score, the bigger the Maximum Likelihood of the random topology model.

TABLE III
MAXIMUM LIKELIHOOD COMPARISON OF RANDOM TOPOLOGY MODELS

$-\log_{10}(\mathrm{ML})$	Poisson-RT	Uniform-RT	ER
IEEE-30	≤ 7.8	≤ 7.6*	≥ 103.23
IEEE-57	≤ 18.3*	≤ 20.4	≥ 217.43
IEEE-118	≤ 106.7	≤ 97.4*	≥ 507.16
IEEE-300	≤ 210.7*	≤ 237.6	≥ 1308.5

* denotes the best ML score in the line.

The results in Table III shows that as long as IEEE power system is considered, models of Poisson-RT and Uniform-RT perform similarly well, much better than ER model. In fact for ER random graph model, when the network size increases, it quickly turns almost impossible to generate a desired topology with specified nodal degree sequence.

6. Conclusion

Adequate and efficient monitoring and communication supports are essentially important to enable an autonomous reconfigurable power system, which responds in an optimal way toward unanticipated disturbances or even catastrophic failures. In search of ways to design effective and efficient communication architecture for a large-scale power grid, it is necessary to study how the required communication capacities scale as the network size grows. For this purpose, the standard practice for power system models to be simulated using a relatively small number of historical test systems is no longer sufficient. A simulation tool, which is able to generate large numbers of realistic power grids with random topologies, becomes greatly useful to assess the communication needs and the performance of the combined electric power and communication network.

The model proposed in this paper to generate random-topology power grids is shown to be effective in two ways – 1) the topological features of the generated power grids approximate closely those of IEEE standard systems and the empirical power system of WSCC and NYISO; 2) the eigenvalues distribution of the generated power grids is very similar to that of IEEE standard system. Between the two varieties of the model, Poisson-RT works better than Uniform-RT in the selected standard systems.

The Maximum Likelihood experiments also verify that the model of Poisson-RT and Uniform-RT can generate a desired topology power grid with a much higher probability than Erdos-Renyi random graph model.

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