Impact of Power Generation Uncertainty on Power System Static Performance

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***Abstract*—The rapid growth in renewable energy resources such as wind and solar generation introduces significant uncertainties on the generation side of power systems. We propose a method to assess whether static state variables, i.e., bus voltage magnitudes and angles, remain within acceptable ranges while the system is subject to uncontrolled disturbances caused by the uncertainty in local installations of renewable resources. The method uses ellipsoidal-shaped sets to bound uncertainty in power injections and the linearized power flow equations to compute approximate bounds on system static states. Numerical results for benchmark 4-bus and 34-bus systems are presented.**

I. Introduction

The push toward environmentally responsible energy usage requires increased penetration of renewable resources of electricity, such as wind and solar generation, into the existing grid. Since these resources are highly intermittent, variable, and difficult to forecast accurately, they present notable uncertainties to the operation of today's power systems. This paper focuses on a particular aspect of the impact of renewable resource penetration on power system static performance and proposes an analytically tractable method, which assesses whether static state variables, i.e., bus voltage magnitudes and angles, remain within acceptable ranges while the system is subject to uncontrolled disturbances caused by the uncertainty in local installations of renewable resources.

Statistical and worst-case analyses are complementary in the assessment of risk involved with power systems operations. We study the worst-case approach, as this provides a guarantee of system security. Uncertain renewable generation is modeled as an unknown quantity constrained between minimum and maximum bounds. We assume the uncertainty introduced by renewable resource penetration is sufficiently small to justify the use of a small-signal approximation around a nominal operating point determined by the forecasted renewable power injection. In our methodology, the uncertainty in generation can be viewed as forecast error, which provides bounds on the variation of the renewable-based generation in the system. These bounds, in conjunction with the linearized model, are used to approximate the set that contains all possible static variable realizations arising from all possible power injections. If this set is contained within the region of static state space defined by system operational requirements, such as minimum and maximum bus voltage values, then we conclude that the renewable generation uncertainty does not have a significant impact on system static performance.

Load flow analysis is the fundamental tool used by power engineers to determine a snapshot of the state of a power system. In a real system, line parameters are subject to modeling inaccuracies and loads contain uncertainties. In order to handle these uncertainties, methods such as probabilistic load flow (PLF) [1], [2] were developed. In PLF, uncertainties in load and generation are modeled as random variables and the output of the power flow computation are probability distributions. A different approach using fuzzy sets to characterize uncertainties in the nodal injections was proposed in [3] and applied to wind generation in [4]. Interval methods [5] provide strict bounds on the solutions of the power flow problem given the input uncertainties lie within a fixed interval. This method has several disadvantages in that the output solution interval may be excessively conservative, containing non-solutions in addition to solutions points. Such a shortcoming is a direct result of bounding the solution interval with a convex hull of the solution points. Ellipsoidal methods applied to network parameter and measurement uncertainties were explored in [6]. This paper extends the work of [6] to incorporate uncertainties on the generation side from renewable resources and uses multiple bounding ellipsoids to approximate the exact bounding set, yielding more accurate bounds than those in [6].

The paper is organized as follows. In Section II, the power flow formulation and the corresponding linearization are described. This is followed by the development of the unknown-but-bounded power injection uncertainty model in Section III. In Section IV, the proposed methodology is used on several benchmark systems, including a 34-bus distribution test case. Finally, concluding remarks are made in Section V.

II. Power System Model

In this section, we derive a linearized static model of the power system from the nonlinear power flow equations. This linearized model is later used in the case studies.

A. Power Flow Formulation

The power flow problem is the computation of voltage magnitude and phase angle at each bus in a power system under balanced three-phase steady-state conditions [7]. For every bus $i = 1, \ldots, n$ in the network, let V_i denote the

voltage magnitude, θ_i the voltage angle, P_i the net real power injection, and Q_i the net reactive power injection. Then,

$$P_i = V_i \sum_{k=1}^{n} V_k \left[G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k) \right], \quad (1)$$

$$Q_i = V_i \sum_{k=1}^{n} V_k \left[G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k) \right], \quad (2)$$

where G_{ik} and B_{ik} are the real and imaginary parts of the (i, k) entry in the network admittance matrix, respectively. Each load bus i has two unknowns, V_i and θ_i, related to both P_i and Q_i equations. Each PV bus i has one unknown, θ_i, corresponding to the P_i relation only. Let m be the number of load buses in the network, then there are $n-m-1$ PV buses. Therefore, in the power flow problem, there are $n+m-1$ equations corresponding to the same number of unknowns.

To consider uncertainty in power injections arising from renewable resources only, we make the proper distinction in the net real and reactive power for each load bus in (1)-(2) as

$$P_i = P_{g_i} - P_{l_i}, \quad (3)$$

$$Q_i = Q_{g_i} - Q_{l_i}, \quad (4)$$

where P_{g_i} and Q_{g_i} are the real and reactive power injections at bus i respectively, P_{l_i} and Q_{l_i} are the real and reactive power demanded at bus i.

B. Model Description

The solution to the power flow equations in (1)-(2) can be rewritten as

$$v = f(x, u), \quad (5)$$

where $f : \mathbb{R}^{n+m-1} \times \mathbb{R}^{n-m-1} \mapsto \mathbb{R}^{n+m-1}$, $x \in \mathbb{R}^{n+m-1}$ represents unknown quantities to be solved for and includes V_i and θ_i for load buses and θ_i for PV buses, $u \in \mathbb{R}^{n-m+1}$ represents the known bus voltages in PV buses, V_1 and θ_1 for the swing bus, and $v \in \mathbb{R}^{n+m-1}$ represents the uncertain inputs and includes P_i for PV buses and P_i and Q_i for PQ buses.

Accounting for the distinction between power generation and consumption at each bus as in (3)-(4), (5) can be re-written as

$$w - u_l = f(x, u), \quad (6)$$

where $w \in \mathbb{R}^{n+m-1}$ represents the vector of real and reactive power injections and $u_l \in \mathbb{R}^{n+m-1}$ represents the vector of real and reactive power demand in the system. We assume the uncertainty in system load is negligible compared to that of renewable generation, since accurate load forecasts are usually available.

In this work, we model the uncertainty in w as unknown-but-bounded and assume that w—power injection from distributed generation—is restricted to some margin around an operating point w^0. Then, w is bounded to some set $\mathcal{W}$ around w^0. Corresponding to $\mathcal{W}$, the set that contains all the possible resulting x is denoted by $\mathcal{R}$. Accordingly, we rewrite the system description in (6) as

$$w = f(x, u) + u_l,$$
$$w \in \mathcal{W}, \; x \in \mathcal{R}. \quad (7)$$

C. Linearized Model

Suppose the system described by (6) is solved with nominal uncertain input $w = w^0$. Let x^0 represent the nominal solution to the power flow problem with inputs (u, u_l, w^0). In other words,

$$w^0 = f(x^0, u) + u_l.$$

Let $x = x^0 + \Delta x$, $w = w^0 + \Delta w$. If the variations in w around w^0 are sufficiently small, then

$$\Delta w \approx \left[\frac{\partial f}{\partial x} \right]_{(x^0, w^0)} \Delta x, \quad (8)$$

where $\partial f / \partial x$ is the Jacobian of the power flow equations. The inverse of this Jacobian matrix evaluated at solution x^0 is guaranteed to exist if the power flow converges to that solution. Thus, near the nominal solution x^0,

$$\Delta x \approx H \Delta w, \quad (9)$$

where

$$H = \left[\left. \frac{\partial f}{\partial x} \right|_{(x^0, w^0)} \right]^{-1}.$$

III. Uncertainty Analysis

In this section, we quantify the uncertainty in the complex bus voltages of a distribution system subject to uncertain power injections arising from renewable resource penetration. In a distribution system, the feeder root is connected to the transmission system at bus 1, which is assumed to be an infinite bus with a constant voltage. All other buses on the distribution feeder are load buses. In our studies, small-scale renewable resources, modeled as negative loads, are installed throughout the distribution system.

A. Unknown-but-Bounded Framework

If the variations in w around w^0 are sufficiently small, we can approximate $\mathcal{R}$ by a set, denoted by $\Delta\mathcal{R}$, that contains all possible Δx in (9). The variations in Δw are bounded by $\Delta\mathcal{W}$, where[1] $\mathcal{W} = w^0 \oplus \Delta\mathcal{W}$. Even though the shape of $\Delta\mathcal{W}$ is arbitrary, it can always be enclosed by an ellipsoid $\Delta\Omega$:

$$\Delta w \in \Delta\mathcal{W} \subseteq \Delta\Omega = \{\Delta w : \Delta w' \Psi^{-1} \Delta w \leq 1\},$$

where Ψ is a positive definite matrix. In this case, $\Delta\mathcal{R}$, is upper bounded by $\Delta\mathcal{X} = \{\Delta x : \Delta x' \Gamma^{-1} \Delta x \leq 1\}$, where Γ, also a positive definite matrix, is obtained by solving

$$\Gamma = H \Psi H',$$

as shown in [8]. Moreover, $\Delta\mathcal{X}$ is the exact set that contains all possible Δx if the input set is, indeed, $\Delta\Omega$.

[1] $\oplus$ denotes the vector sum of the vector w^0 and the set $\Delta\mathcal{W}$.

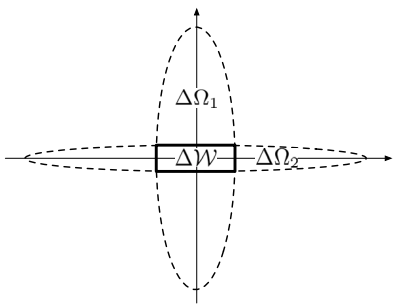


Fig. 1: Approximation of the input space as intersection of ellipsoids.

The set $\Delta\mathcal{W}$ is usually a symmetrical polytope, i.e., each entry in w is assumed to lie within some interval. In this paper, each entry in w represents the power injected at a bus originating from renewable resources, such as a residential solar panel installation. A symmetrical polytope can always be approximated to a high degree of precision by the intersection of a family of ellipsoids, each of which is tight to the polytope in a specific direction, as shown in Fig. 1. When several ellipsoids are used to bound the disturbance set $\Delta\mathcal{W}$, the set $\Delta\mathcal{X}$ can be computed for each of the ellipsoids bounding $\Delta\mathcal{W}$, and then the intersection of the resulting bounding sets (for each bounding ellipsoid) yields a high-fidelity approximation of the exact set $\Delta\mathcal{R}$. In this case, $\Delta\mathcal{W} \subseteq \bigcap_i \Delta\Omega_i$, where

$$\Delta w \in \Delta\Omega_i = \{\Delta w : \Delta w' \Psi_i^{-1} \Delta w \leq 1\},$$

each of which bounds $\Delta\mathcal{W}$. The resulting set that bounds Δx for each ellipsoid $\Delta\Omega_i$, denoted by $\Delta\mathcal{X}_i$, is solved as

$$\Delta\mathcal{X}_i = \{\Delta x : \Delta x' \Gamma_i^{-1} \Delta x \leq 1\}.$$

Thus, $\Delta x \in \Delta\mathcal{R} \subseteq \bigcap_i \Delta\mathcal{X}_i$.

B. Performance Requirements

Static performance requirements on distribution systems generally consist of constraints in the form of interval ranges on bus voltages. For example, bus voltage magnitudes are generally required to be between 0.95 p.u. and 1.05 p.u. These requirements constrain the excursion of the state vector x around x_0 to some region of the state space Φ defined by the symmetric polytope

$$\Phi = \{x : |\pi_i'(x - x_0)| \leq 1 \quad \forall i = 1, 2, \ldots, p\}.$$

The computation of the set that contains all possible x, given uncertainty in power injections, allows us to determine whether the system violates performance requirements that impose maximum deviations of system variables. In fact, verifying that the system meets performance requirements for any $w \in \mathcal{W}$ is equivalent to checking that $\Delta\mathcal{R} \subseteq \Phi$.

IV. Case Studies

In this section, we illustrate the concepts developed in this paper by presenting the results of several benchmark systems. The benchmark systems are taken from the IEEE PES Distribution System Analysis Subcommittee, which are modified to include power injection at certain buses. In these systems, the power base is 100 kVA and voltage base is 4.16 kV.

TABLE I: Two-bus system nominal power flow solution.

w^0		u_l		u		x^0	
$P_{g_2}^0$	$Q_{g_2}^0$	$P_{l_2}^0$	$Q_{l_2}^0$	V_1	θ_1	V_2^0	θ_2^0
0.3	0	0.8	0.5	0.97	0°	0.9543	-0.3095°

Example 1 (Two-bus model): This simple example is illustrated in Fig. 2, where $R = 0.01$ p.u. and $X = 0.02$ p.u.. We assume there is a wind turbine or an aggregate of several wind turbines installed at bus 2, which is forecasted to inject 0.30 p.u. real power. In addition, there is uncertainty in the power injected at bus 2, P_{g_2}, and the power demanded at bus 2, P_{l_2}. The admittance matrix for this network is

$$\overline{\mathbf{Y}} = \begin{bmatrix} 20 - j40 & -20 + j40 \\ -20 + j40 & 20 - j40 \end{bmatrix},$$

and the power flow equations are

$$P_2 = V_2 V_1 \left[-20\cos(\theta_2 - \theta_1) + 40\sin(\theta_2 - \theta_1)\right] + 20V_2^2,$$
$$Q_2 = V_2 V_1 \left[-20\sin(\theta_2 - \theta_1) - 40\cos(\theta_2 - \theta_1)\right] + 40V_2^2,$$

where $P_2 = P_{g_2} - P_{l_2}$ and $Q_2 = Q_{g_2} - Q_{l_2}$. The nominal solution is shown in Table I, where all numerical values are per unit unless otherwise indicated. The linearized system is

$$\begin{bmatrix} \Delta P_{g_2} \\ \Delta Q_{g_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial Q_2}{\partial \theta_2} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix}\Bigg|_{(V_2^0, \theta_2^0)} \begin{bmatrix} \Delta\theta_2 \\ \Delta V_2 \end{bmatrix},$$

where

$$\frac{\partial P_2}{\partial \theta_2} = 20V_1V_2\sin(\theta_2 - \theta_1) + 40V_1V_2\cos(\theta_2 - \theta_1),$$
$$\frac{\partial P_2}{\partial V_2} = V_1[-20\cos(\theta_2 - \theta_1) + 40\sin(\theta_2 - \theta_1)] + 40V_2,$$
$$\frac{\partial Q_2}{\partial \theta_2} = -20V_1V_2\cos(\theta_2 - \theta_1) + 40V_1V_2\sin(\theta_2 - \theta_1),$$
$$\frac{\partial Q_2}{\partial V_2} = V_1\left[-20\sin(\theta_2 - \theta_1) - 40\cos(\theta_2 - \theta_1)\right] + 80V_2.$$

Thus, evaluating the Jacobian about nominal power flow solution, we obtain

$$\begin{bmatrix} \Delta P_{g_2} \\ \Delta Q_{g_2} \end{bmatrix} = \begin{bmatrix} 36.925 & 18.5614 \\ -18.7125 & 37.6467 \end{bmatrix} \begin{bmatrix} \Delta\theta_2 \\ \Delta V_2 \end{bmatrix}.$$

Suppose wind turbines are installed at bus 2 at a rated capacity of 0.45 p.u. and they are forecasted to produce 0.3

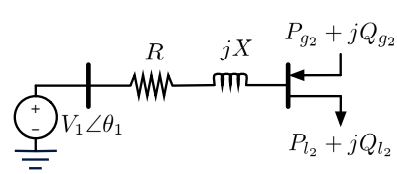


Fig. 2: Two-bus example.

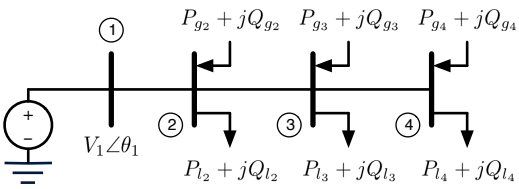


Fig. 3: Modified 4-bus feeder system with renewable power injection.

p.u. real power with a forecast error of ± 0.1 p.u. and no reactive power. Let $\Delta w = [\Delta P_{g_2}, \Delta Q_{g_2}]'$, then the input disturbance is bounded by $\Delta\Omega = \{\Delta w : \Delta w' \Psi^{-1} \Delta w \leq 1\}$, where

$$\Psi = \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0 \end{bmatrix}.$$

In this case, $\Delta\mathcal{W} = \Delta\Omega$, since there is only uncertainty in one dimension. Let $\Delta x = [\Delta\theta_2, \Delta V_2]'$, then $\Delta\mathcal{R} = \Delta\mathcal{X} = \{\Delta x : \Delta x' \Gamma^{-1} \Delta x \leq 1\}$, where

$$\Gamma = \begin{bmatrix} 0.0022^2 & 0.0015^2 \\ 0.0015^2 & 0.0011^2 \end{bmatrix}.$$

We project the set $\Delta\mathcal{R}$ onto the V_2-axis to obtain worst-case deviations of the variable as ± 0.0011 p.u. about its nominal operating value 0.9543 p.u. Therefore, we conclude that with 0.3 ± 0.1 p.u. renewable power injection at bus 2, its voltage magnitude lies in the range $[0.9532, 0.9554]$, which is within voltage constraints. ■

TABLE II: Four-bus system nominal power flow solution

w^0	$P^0_{g_2}$ 0.4	$Q^0_{g_2}$ 0	$P^0_{g_3}$ 0.3	$Q^0_{g_3}$ 0	$P^0_{g_4}$ 0.5	$Q^0_{g_4}$ 0
u_l	P_{l_2} 0.8	Q_{l_2} 0.25	P_{l_3} 0.5	Q_{l_3} 0.1	P_{l_4} 0.9	Q_{l_4} 0.5
x^0	V^0_2 0.987	θ^0_2 $-0.124°$	V^0_3 0.972	θ^0_3 $-0.273°$	V^0_4 0.965	θ^0_4 $-0.302°$

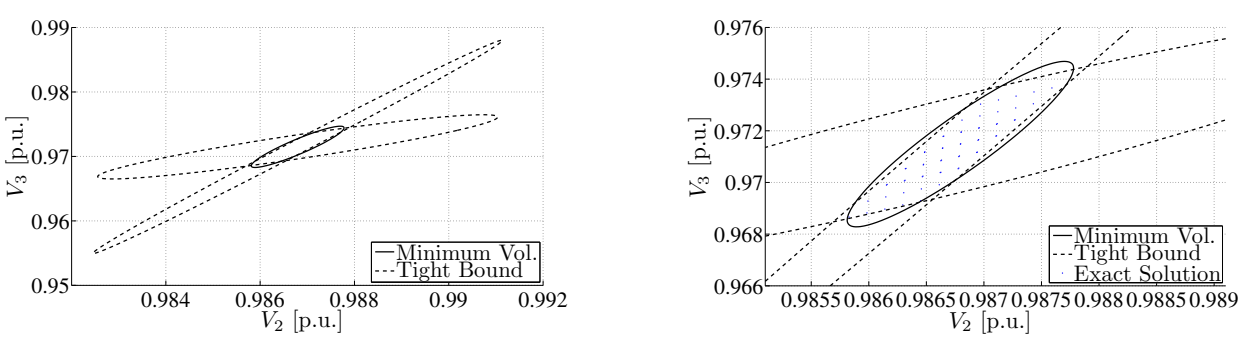
A. 4-*bus System*

This test feeder system is shown in Fig. 3. Here, bus 1 is the slack bus with voltage $0.995\angle 0°$ and is connected to a substation. The operating point as dictated by the power flow solution is shown in Table II. We assume distributed renewable resources installed at buses 2, 3, and 4 are forecasted to inject 0.4, 0.3, and 0.5 p.u. real power, at their respective buses. The renewable power injections are assumed to vary between $\pm 20\%$ of the forecast values. None of the renewable resources provide any reactive power, and there is no uncertainty associated with reactive power injections.

The procedure described in Section III is used on the 4-bus system, and the variations in V_2 and V_3 are presented in Fig. 4. We tightly bound the input disturbance space as the intersection of three ellipsoids centered around the operating point as in Table II: $\Delta\Omega_1$ and $\Delta\Omega_2$, which tightly bound $\Delta\mathcal{W}$ in two orthogonal directions, and $\Delta\Omega_3$, which is a minimum volume ellipsoid that circumscribes $\Delta\mathcal{W}$. In Fig. 4(a), the ellipsoids depicted in dashed lines are generated from sets $\Delta\Omega_1$ and $\Delta\Omega_2$, and the ellipsoid with the solid trace is generated from $\Delta\Omega_3$. The exact set $\Delta\mathcal{R}$ is bounded by the intersection of the ellipsoids in Fig. 4(a), a magnified view of which is shown in Fig. 4(b). For comparison, we also obtain solutions of the nonlinear power flow relations by sampling the power injection space, which are depicted as points in Fig. 4(b). We see that the intersection of the resultant ellipsoidal sets obtained from the linearized power flow equations are, indeed, an accurate bound to the nonlinear solutions for $\pm 20\%$ uncertainty. In fact, we find that the linearized set approximation is valid for up to $\pm 50\%$ input uncertainty.

B. 34-*bus System*

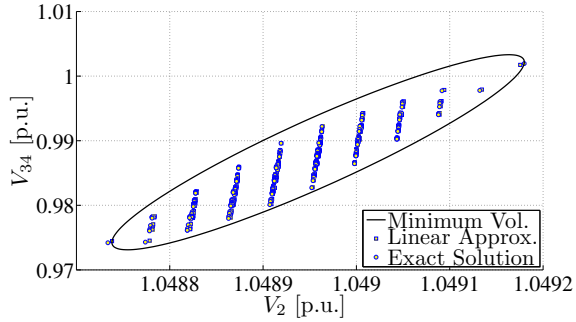
The one-line diagram and complete description of this radial feeder system can be found in [9]. We assume that distributed renewable resources are installed at buses 3, 7, 10, 15, 18, 23, 27, 29, 30, and 34, and their power outputs vary between $\pm 50\%$ of the forecasted values. Our methodology is applied to this test system and select results are shown in Fig. 5.



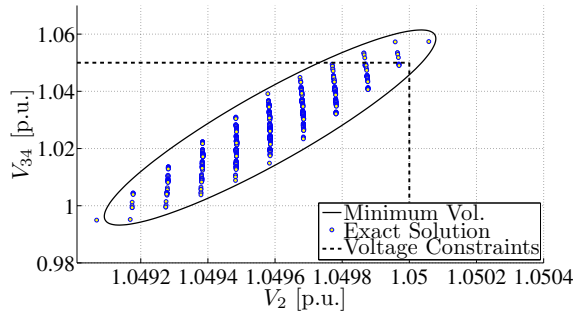
(a) Ellipsoidal bounds of the linearized 4-bus test system, where the set containing all possible voltage magnitude values is obtained as the intersection.

(b) Nonlinear power flow solutions using input bounds in comparison with bounding set obtained for the linearized 4-bus test system.

Fig. 4: Bounds on voltage magnitudes at buses 2 and 3 for the 4-bus test system.



(a) Comparison between the bounding ellipsoid, exact solutions of the linearized power flow relations, and the exact solutions of the nonlinear power flow. Each renewable resource provides 0.4 p.u. real power injection at their respective buses.



(b) Comparison between the bounding ellipsoid and the exact solutions of the nonlinear power flow. Each renewable resource provides 1.0 p.u. real power injection at their respective buses.

Fig. 5: Ellipsoidal bound of the linearized 34-bus test system for voltage magnitudes at buses 2 and 34, with renewable resources installed at 10 buses and power injection uncertainty of $\pm 50\%$ of the forecast.

For simplicity, we assume all distributed resources are forecasted to output the same amount of real power. We bound the power injection space with a minimum volume ellipsoid that circumscribes $\Delta\mathcal{W}$. Fig. 5(a) depicts results for the case in which each resource is forecasted to output 0.4 p.u., while Fig. 5(b) shows results for 1.0 p.u. injection. We sample the power injection and obtain the corresponding exact solutions to the linearized power flow as well as those to the original nonlinear power flow relations, depicted as squares and circles, respectively, in Fig. 5(a). As expected, the resulting ellipsoidal bounding set contains all the linearized power flow solutions with the extrema coinciding with the edge of the ellipsoid. The linearization is fairly accurate in this system; only one nonlinear solution corresponding to the lower extreme point of the input sample space is not contained in the linearized solution set.

With the power injection and uncertainty levels represented in Fig. 5(a), we do not detect any voltage magnitude violations. In comparison, in Fig. 5(b), for higher levels of renewable penetration, we see that a portion of the input space maps to a region in the solution state space that violates voltage constraints, which are depicted with dashed lines. This conclusion is, again, verified by sampling the input space and computing the corresponding exact nonlinear power flow solution.

V. Concluding Remarks

This paper proposes a method for the assessment of the impact of uncertain distributed generation on power system static performance. The proposed method determines whether system variables remain within prescribed ranges as dictated by operational requirements. We formulate a set-theoretic method, which provides a guarantee of system security, to obtain the worst-case deviations of static system states.

We approximate the input uncertainty as the intersection of several ellipsoids and calculate the set that encloses all deviations of the system static variables. As shown in the test cases, the bounding set obtained with our method matches closely to those obtained from repeatedly solving the nonlinear power flow for different power injections. Our method is computationally attractive since linear approximations are used and only several ellipsoids are required to establish an accurate approximation to the actual bounding set. In contrast, a Monte Carlo type of simulation requires sampling the input uncertainty set many times in addition to calculating the nonlinear power flow for each sample point. Another advantage of our method is its versatility: it can be used for uncertainty in real and reactive power supply and demand alike.

Further work includes an analysis of the limits of the small-signal approximation to the power flow relations. Another aspect to be investigated is the scalability of the proposed method; the results from the 4-bus and 34-bus test systems are encouraging in this regard.

References

[1] B. Borkowska, "Probabilistic load flow," *IEEE Transactions on PAS*, vol. 93, no. 3, pp. 752–759, Aug. 1974.

[2] P. Sauer, "A generalized stochastic power flow algorithm," in *IEEE 1978 Power Engineering Society Summer Meeting*, 1978, pp. 544–9.

[3] A. Dimitrovski and K. Tomsovic, "Boundary load flow solutions," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 348–355, Feb. 2004.

[4] L. Hong, L. Shi, L. Yao, Y. Ni, and M. Bazargan, "Study on fuzzy load flow with consideration of wind generation uncertainties," in *Transmission and Distribution Conference and Exposition: Asia and Pacific, T and D Asia*, 2009.

[5] Z. Wang and F. Alvarado, "Interval arithmetic in power flow analysis," *IEEE Transactions on Power Systems*, vol. 7, no. 3, pp. 1341–1349, Aug. 1992.

[6] A. Saríc and A. Stankovíc, "Ellipsoidal approximation to uncertainty propagation in boundary power flow," *Power Systems Conference and Exposition, 2006.*, 2006.

[7] J. Glover, M. Sarma, and T. Overbye, *Power System Analysis and Design*. Cengage Learning, 2008.

[8] F. Schweppe, *Uncertain Dynamic Systems*. Englewood Cliffs, NJ: Prentice-Hall Inc., 1973.

[9] W. Kersting, "Radial distribution test feeders," *IEEE Transactions on Power Systems*, vol. 6, no. 3, pp. 975 –985, aug 1991.