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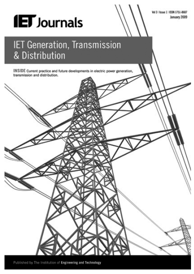

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Mixed power flow analysis using AC and DC models

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Abstract: Steady-state power flow analysis for large-scale networks requires substantial computation because of the extensive interconnections among power systems. Traditionally, network equivalent techniques have been used to reduce computational demand by eliminating the external system. These techniques, then, cannot reflect changes in the external system. This study presents a mixed approach with ac and dc power flow models for power flow analysis to decrease computational complexity and capture variations in the external system. A high level of accuracy in the targeted central part of the system is achieved using the detailed ac model. The less detailed dc model is used to reduce computational requirements and still reflect changes in the external system. Case studies with the IEEE 118-bus system are provided to compare performance among the proposed, the ac and the dc models.

Nomenclature

$\overline{\boldsymbol{V_k}} = \boldsymbol{V_k} \angle \boldsymbol{\theta_k}$	complex voltage at bus k
V_k	voltage magnitude at bus k
θ_k	voltage phase angle at bus k
P_k	net injected real power at bus k
Q_k	net injected reactive power at bus k
G_{km}	real part of admittance matrix element $\overline{\boldsymbol{Y}}_{km}$
B_{km}	imaginary part of admittance matrix element $\overline{\boldsymbol{Y}}_{km}$
$\underline{\boldsymbol{J}}$	Jacobian matrix
f_k^p, f_k^q	mismatch equations of real and reactive power injection at bus k, respectively
Superscripts ^{sp} and _	denote specified value and matrix, respectively

1 Introduction

The function of an electric power system during normal operating conditions is to supply power to customers while maintaining voltage and frequency within predetermined limits. It is essential to know the voltages at each bus as well as the power flows through a transmission network in order to obtain complete understanding of the power system. Power flow analysis can provide this information, which can then be used to simulate the effectiveness of a future power system expansion plan. The simulation checks component overloading during peak load periods. Power flow analysis also allows system operators and planners to prepare for unpredictable contingency events such as loss of a generating unit or a transmission line outage. In addition, real-time results from periodically executed online power flow study are used to correct the power factor by compensating the reactive power and to allocate optimal generation to minimise transmission line losses and generation cost. Hence, power flow analysis is fundamental for operating and planning power systems.

The most accurate approach for power flow analysis is to model the electric power system with the classic power flow equations (ac model). The ac model is formulated by a set of non-linear algebraic equations. An iterative algorithm is needed to solve it [1, 2]. Several problems are associated with the ac model. The non-linear equations may not converge when a good initial guess of the solution is not available [3]. High convergence reliability can be achieved with time consuming synthetic dynamics power flow method, which adds artificial dynamic equations to the non-linear equations [4]. The ac model is computationally expensive, especially when contingency analysis is considered.

A number of approximate models and different approaches to the power flow problem to improve performance have been studied. Some approximate models using physical properties of power systems, such as the decoupled and the dc power flow models, are the most commonly used analysis techniques in power systems [5–7]. These have faster solutions and simplicity. Examples include contingency analysis, transmission planning and market applications. Modifications for fast solution and less convergence difficulties of the decoupled model have been proposed [8, 9]. The quadratic power flow model provides faster convergence by using quadratic equations ideally suited to Newton's method [10]. In addition, considering that uncertainty is always present in power systems, its incorporation into the solution process has been proposed [11, 12]. Either probabilistic power flow or fuzzy power flow model is used, depending on how one expresses the uncertainty in the system.

Power systems have been required to operate more efficiently and economically since the deregulation of the power industry. To accomplish these objectives, it is important for power systems control centres to be able to analyse system states accurately and quickly. However, such analysis is a computationally demanding problem in large modern interconnected power systems, particularly for long-term simulations. To secure energy delivery systems, it is crucial to develop rapid and precise analysis method in order to have real-time situational awareness [13].

Many efforts have been made to speed up the power system analysis. Parallel computers using multiple processing units can achieve fast simulation without simplifying the transmission network [14]. Traditionally, network equivalent techniques have been used to reduce computational requirements [15]. Deckmann *et al.* [16, 17] give a comprehensive overview of classic methods to derive equivalent networks and their performance results in terms of accuracy, convergence and conditioning. An analytic study based on practical experience and its application for security assessment can be found in [18–20]. Network equivalent techniques partition the electric network into the internal system, external system and a group of boundary buses that divide the external system from the internal system. The size of the power network is reduced by eliminating the external system, whereas the internal system is unchanged. The effect of the external system on the internal one is included by adding real and reactive power flows to the boundary buses. In practical power systems, the internal system usually denotes the monitored part of the interconnected power system and is the area of interest of a regional utility.

However, it should be understood that the network equivalent approach is practical only for applications without variations in the external system or when knowledge of voltage states of the entire or the internal system is already available [16, 18, 19]. This approach cannot capture changes in bus injections and network status of the external system because the external system was previously eliminated. As a result, the equivalent network needs to be updated whenever an alteration to the external system occurs. The approach needs to have the solved power flow case for boundary matching: the net power flows between the unreduced and the reduced networks must be an exact match in the boundary buses. Therefore errors may be introduced when the solved power flow case is unknown.

This paper presents an approach focused on reducing computational requirements for power flow studies, while being able to take into account bus injections and network status in the external system. The proposed method combines the ac and the dc power flow models. To achieve a high level of accuracy in the area of interest, power flow problems are formulated with the detailed ac model in the internal system. To reduce computational expense and reflect external variations, problems are solved with the less detailed dc model in the remaining system.

This paper is organised as follows: Section 2 presents a brief analytic basis for power flow analysis. The proposed approach is presented in Section 3. Section 4 illustrates simulation results with the IEEE 118-bus system. The conclusion is presented in Section 5.

2 Power flow analysis

The basic formulation and solution of the power flow equations are presented briefly in this section.

2.1 AC power flow model

The formulation of the ac power flow equations begins with nodal analysis. The power balance equations are

$$P_k = V_k \sum_{m=1}^{N} V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] \quad (1)$$

$$Q_k = V_k \sum_{m=1}^{N} V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] \quad (2)$$

The real and reactive power balance equations in (1) and (2), respectively, are expressed with four variables: voltage magnitude, voltage phase angle, and net real and net reactive power injections. Two of the four variables at each bus are known, depending on bus type. The remaining variables can be obtained by solving a set of non-linear power balance equations. In order to solve for the unknowns in a power system, there must be the same number of equations as unknowns. The power balance equations at each bus are used. These equations can be formulated depending on bus type.

- Load bus (PQ bus)

$$P_k^{sp} = V_k \sum_{m=1}^{N} V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] \quad (3)$$

$$Q_k^{sp} = V_k \sum_{m=1}^{N} V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] \quad (4)$$

- Generator bus (PV bus)

$$P_k^{sp} = V_k \sum_{m=1}^{N} V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] \quad (5)$$

A resulting set of non-linear equations can be solved with Newton–Raphson (NR) method.

2.2 DC power flow model

The dc power flow model greatly simplifies the ac model with the following assumptions:

1. Voltage magnitudes on all buses are 1 pu.
2. Voltage angle differences are small: $\sin(\theta_k - \theta_m) \simeq \theta_k - \theta_m$, $\cos(\theta_k - \theta_m) \simeq 1$.
3. Line resistance is negligible: $G_{km} \simeq 0$.
4. Reactive power injections on all buses are ignored.

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Hence, the real power balance equation in (1) can be approximated as

$$P_k \simeq \sum_{m=1}^{N} B_{km}(\theta_k - \theta_m) \quad (6)$$

The dc model has computational advantages over the ac model. First, its equation set is just about half of the AC model, because it considers only the real power injections. Second, the dc model requires no iteration. Third, because the $\boldsymbol{B}$ matrix is independent of the states, only one factorisation is necessary. Therefore to find voltage states, the dc model is about ten times faster than the ac model [21]. Although the dc model is inherently approximate and may introduce error in power flow analysis, several previous attempts show that results using the dc model are reasonable and dc line power flows are, on average, offset by a few percentage points, as compared to the ac model [7, 21, 22].

3 Proposed methodology

The proposed approach formulates the power flow problem by combining the ac with the dc models in order to reduce the computational expenses and take the bus injections and network status in the external system into account. Power flow equations in the internal system, which require accurate solution, are formulated with the ac model and those in the external system are done with the dc model for a faster but less-detailed solution. Then, the reduced set of non-linear equations is solved with the Newton-Raphson (NR) method. Fig. 1 shows the procedure.

3.1 Internal/external system and boundary buses

The power system can be divided into three mutually exclusive subsystems dependent on the area of interest, which is called the 'internal system'. The internal system is connected to neighbouring systems, called the 'external system'. A group of buses in the external system, which have a connection with a bus in the internal system are called 'boundary buses'. Fig. 2 defines these three subsystems.

3.2 Assumption of system information

It is assumed that all necessary information (net real and net reactive power injections for a Load Bus (PQ) bus and net real power injection and voltage magnitude for a bus generator bus (PV)) is given in the internal system, whereas only net real power injection is given in the external system and at the boundary buses. In addition, the complete network data are assumed to be known.

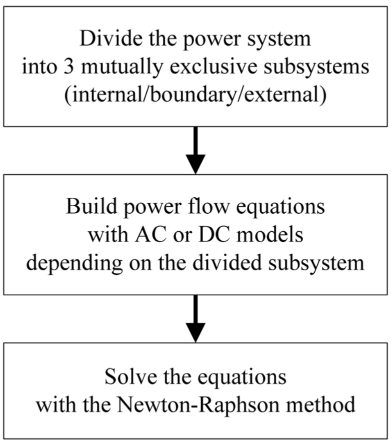

Fig. 1 *Procedure for the approach*

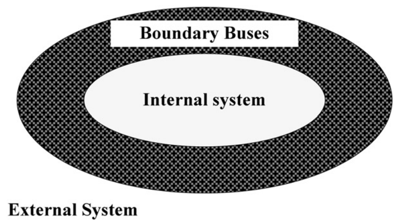

Fig. 2 *Schematic representation of internal, boundary buses and external system*

3.3 Boundary assumption

The effect of the external system on the internal system through the boundary buses should be considered in order to attain a high level of accuracy in the internal system. However, the information given at the boundary bus is assumed to have only the net real power injection. Therefore a proper guess for reactive power injection or voltage magnitude at the boundary buses is required. The voltage magnitude difference between two buses on a transmission line is usually around 1–2%. Each boundary bus has at least one connection to the internal system bus whose voltage magnitude would be accurate because of a set of non-linear equations with all given information. Thus, the best guess for the boundary bus voltage magnitude would be to use that of the internal system. In addition, for a precise voltage phase-angle solution, non-linear real power balance equations are built at the boundary buses. In other words, the approach assumes that the boundary buses are considered to be a PV bus with the voltage magnitude of a connected internal system bus without regard to bus type.

3.4 Power flow problem formulation

Depending on the subsystem, the power flow equations are formulated with AC or models as follows:

1. Internal system

   PQ bus: P, Q non-linear equations using (3) and (4);
   PV bus: P non-linear equation using (5).

2. Boundary buses

   PQ/PV bus: P non-linear equation using (3) with voltage magnitude of a connected internal system bus.

3. External system

   PQ/PV bus: P linear equation using (6).

3.5 Power flow problem solution

The developed power flow problem is still a set of non-linear equations, even though a set of linear equations is formulated

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in the external system. It can be solved with the NR method as follows:

- Build the Jacobian matrix

$$\underline{J} = \begin{bmatrix} \dfrac{\partial \underline{f}^p_{\text{ext}}}{\partial \underline{\theta}_{\text{ext}}} & \dfrac{\partial \underline{f}^p_{\text{ext}}}{\partial \underline{\theta}_{\text{boundary}}} & 0 & 0 \\ \dfrac{\partial \underline{f}^p_{\text{boundary}}}{\partial \underline{\theta}_{\text{ext}}} & \dfrac{\partial \underline{f}^p_{\text{boundary}}}{\partial \underline{\theta}_{\text{boundary}}} & \dfrac{\partial \underline{f}^p_{\text{boundary}}}{\partial \underline{\theta}_{\text{int}}} & \dfrac{\partial \underline{f}^p_{\text{boundary}}}{\partial \underline{V}_{\text{int}}} \\ 0 & \dfrac{\partial \underline{f}^p_{\text{int}}}{\partial \underline{\theta}_{\text{boundary}}} & \dfrac{\partial \underline{f}^p_{\text{int}}}{\partial \underline{\theta}_{\text{int}}} & \dfrac{\partial \underline{f}^p_{\text{int}}}{\partial \underline{V}_{\text{int}}} \\ 0 & \dfrac{\partial \underline{f}^q_{\text{int}}}{\partial \underline{\theta}_{\text{boundary}}} & \dfrac{\partial \underline{f}^q_{\text{int}}}{\partial \underline{\theta}_{\text{int}}} & \dfrac{\partial \underline{f}^q_{\text{int}}}{\partial \underline{V}_{\text{int}}} \end{bmatrix} \tag{7}$$

- The iterations are continued until the stopping criterion is satisfied.

$$\underline{J}^{(i)} \begin{bmatrix} \Delta\underline{\theta}^{(i)}_{\text{ext}} \\ \Delta\underline{\theta}^{(i)}_{\text{boundary}} \\ \Delta\underline{\theta}^{(i)}_{\text{ext}} \\ \Delta\underline{V}^{(i)}_{\text{ext}} \end{bmatrix} = -\begin{bmatrix} \underline{f}^p_{\text{ext}}(\underline{\theta}^{(i)}, \underline{V}^{(i)}) \\ \underline{f}^p_{\text{boundary}}(\underline{\theta}^{(i)}, \underline{V}^{(i)}) \\ \underline{f}^p_{\text{int}}(\underline{\theta}^{(i)}, \underline{V}^{(i)}) \\ \underline{f}^q_{\text{int}}(\underline{\theta}^{(i)}, \underline{V}^{(i)}) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \underline{\theta}^{(i+1)}_{\text{ext}} \\ \underline{\theta}^{(i+1)}_{\text{boundary}} \\ \underline{\theta}^{(i+1)}_{\text{int}} \\ \underline{V}^{(i+1)}_{\text{int}} \end{bmatrix} = \begin{bmatrix} \underline{\theta}^{(i)}_{\text{ext}} \\ \underline{\theta}^{(i)}_{\text{boundary}} \\ \underline{\theta}^{(i)}_{\text{int}} \\ \underline{V}^{(i)}_{\text{int int}} \end{bmatrix} + \begin{bmatrix} \Delta\underline{\theta}^{(i)}_{\text{ext}} \\ \Delta\underline{\theta}^{(i)}_{\text{boundary}} \\ \Delta\underline{\theta}^{(i)}_{\text{int}} \\ \Delta\underline{V}^{(i)}_{\text{int}} \end{bmatrix} \tag{9}$$

3.6 Example

When the simple power system in Fig. 3 is given and bus 1 is assumed to be a slack bus, the mixed approach can be applied as follows:

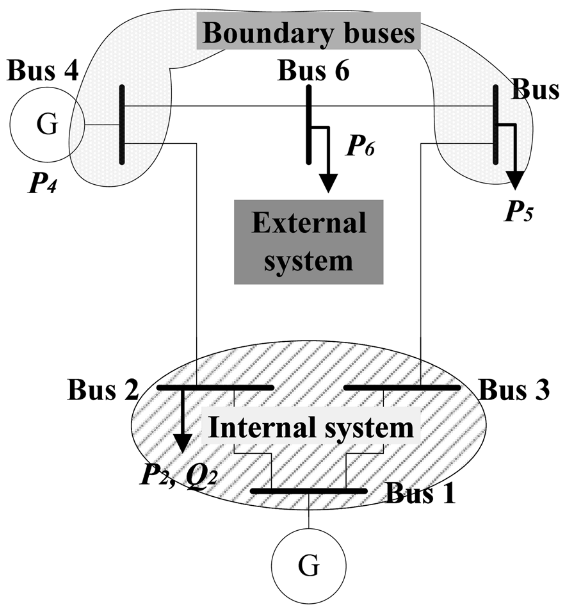

Fig. 3 *Six-bus system*

- *Step 1*: System division

  If it is assumed that the internal system consists of buses 1, 2 and 3, the boundary buses, which have a connection to a bus in the internal one, are buses 4 and 5 and the remaining bus, 6, is the external system.

- *Step 2*: Formulate power flow equations

  Internal system : P, Q non-linear equations at buses 2 and 3

$$f_2^p = V_2 \sum_{m=1}^{N} V_m[G_{2m}\cos(\theta_2 - \theta_m) + B_{2m}\sin(\theta_2 - \theta_m)] + P_2 = 0$$

$$f_2^q = V_2 \sum_{m=1}^{N} V_m[G_{2m}\sin(\theta_2 - \theta_m) - B_{2m}\cos(\theta_2 - \theta_m)] + Q_2 = 0$$

$$f_3^p = V_3 \sum_{m=1}^{N} V_m[G_{3m}\cos(\theta_3 - \theta_m) + B_{3m}\sin(\theta_3 - \theta_m)] = 0$$

$$f_3^q = V_3 \sum_{m=1}^{N} V_m[G_{3m}\sin(\theta_3 - \theta_m) - B_{3m}\cos(\theta_3 - \theta_m)] = 0$$

Boundary buses : P non-linear equation at buses 4 and 5 with voltage magnitude of buses 2 and 3, respectively

$$f_4^p = V_2 \sum_{m=1}^{N} V_m[G_{4m}\cos(\theta_4 - \theta_m) + B_{4m}\sin(\theta_4 - \theta_m)] - P_4 = 0$$

$$f_5^p = V_3 \sum_{m=1}^{N} V_m[G_{5m}\cos(\theta_5 - \theta_m) + B_{5m}\sin(\theta_5 - \theta_m)] + P_5 = 0$$

External system : P linear equation at bus 6

$$f_6^p = \sum_{m=1}^{N} B_{6m}(\theta_6 - \theta_m) - P_6 = 0$$

The total number of system unknowns with the mixed approach is seven, corresponding to the number of formulated equations. Therefore the set of non-linear and linear equations can be solved with the NR method. In contrast to the six-bus example, the dimensions of the external system in practical large-scale power systems are much larger than for the internal one, and thus compared with the set of non-linear equations using the ac model alone, the reduced set of equations can be solved quickly.

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Table 1 Computational benefits from the mixed approach with N buses system

		Ratio of internal to external buses			
		1:5		1:100	
		AC	Mixed	AC	Mixed
LU factorisation	number of operations for the first iteration	$[1/6 \times 2 \times (N-1)]^{1.58} + [5/6 \times 2 \times (N-1)]^{1.58} = 2.42(N-1)^{1.58}$	$[1/6 \times 2 \times (N-1)]^{1.58} + 1/8 \times [5/6 \times 2 \times (N-1)]^{1.58} = 0.46(N-1)^{1.58}$	$[1/101 \times 2 \times (N-1)]^{1.58} + [100/101 \times 2 \times (N-1)]^{1.58} = 2.95(N-1)^{1.58}$	$[1/101 \times 2 \times (N-1)]^{1.58} + 1/8 \times [100/101 \times 2 \times (N-1)]^{1.58} = 0.37(N-1)^{1.58}$
	percentage of operation required	100%	19%	100%	12.5%
	number of operations for the rest iteration	$3 \times 2.42(N-1)^{1.58} = 7.26(N-1)^{1.58}$	$3 \times [1/6 \times 2 \times (N-1)]^{1.58} = 0.53(N-1)^{1.58}$	$3 \times 2.95(N-1)^{1.58} = 8.85(N-1)^{1.58}$	$3 \times [1/101 \times 2 \times (N-1)]^{1.58} = 0.006(N-1)^{1.58}$
	total	$9.68(N-1)^{1.58}$	$0.99(N-1)^{1.58}$	$11.8(N-1)^{1.58}$	$0.38(N-1)^{1.58}$
	percentage of operation required	100%	10.2%	100%	3.2%
forward and back substitution	number of operations for the first iteration	$[1/6 \times 2 \times (N-1)]^{1.29} + [5/6 \times 2 \times (N-1)]^{1.29} = 2.18(N-1)^{1.29}$	$[1/6 \times 2 \times (N-1)]^{1.29} + 1/4 \times [5/6 \times 2 \times (N-1)]^{1.29} = 0.73(N-1)^{1.29}$	$[1/101 \times 2 \times (N-1)]^{1.29} + [100/101 \times 2 \times (N-1)]^{1.29} = 2.42(N-1)^{1.29}$	$[1/101 \times 2 \times (N-1)]^{1.29} + 1/4 \times [100/101 \times 2 \times (N-1)]^{1.29} = 0.61(N-1)^{1.29}$
	percentage of operation required	100%	33.5%	100%	25.2%
	number of operations for the rest iteration	$3 \times 2.18(N-1)^{1.29} = 6.54(N-1)^{1.29}$	$3 \times [1/6 \times 2 \times (N-1)]^{1.29} = 0.73(N-1)^{1.29}$	$3 \times 2.42 \times (N-1)^{1.29} = 7.26(N-1)^{1.29}$	$3 \times [1/101 \times 2 \times (N-1)]^{1.29} = 0.02(N-1)^{1.29}$
	total	$8.72(N-1)^{1.58}$	$1.46(N-1)^{1.58}$	$9.68(N-1)^{1.58}$	$0.63(N-1)^{1.58}$
	percentage of operation required	100%	16.7%	100%	6.5%

3.7 Computational benefits

The proposed approach can reduce the computational requirements with the fast dc model in the external system. Estimation of the computational benefit from the mixed approach is explored by calculating the number of operations for LU factorisation and forward/back substitution requiring the solution of $\underline{\boldsymbol{Ax}} = \underline{\boldsymbol{b}}$, where $\underline{\boldsymbol{A}}$ is the $N \times N$ non-singular sparse matrix. According to [23], when each bus is assumed to have, on average, three branches, the computational complexity for LU factorisation and forward/back substitution for solving $\underline{\boldsymbol{Ax}} = \underline{\boldsymbol{b}}$ is assumed to grow as $N^{1.58}$ and $N^{1.29}$, respectively. The linearised matrix equation from the NR method at each iteration can be considered as $\underline{\boldsymbol{Ax}} = \underline{\boldsymbol{b}}$ and the dimension of the Jacobian matrix is linearly proportional to the number of equations, which are formulated with power flow models.

To simplify the calculations, the number of operations is evaluated assuming that all buses except the slack bus are PQ bus and the number of boundary buses is small enough to be neglected. Most modern power flow code with the ac model treats the Jacobian as a matrix of 2 by 2 blocks. However, when the dc model is applied, the block is replaced with 1 × 1 block. Therefore the dc model is eight times faster for the LU factorisation and four times faster for the forward/back substitution than the ac model. This allows the mixed approach to be faster. Table 1 shows the operations required for both the ac model and the mixed approach for any system having N buses with ratios of internal to external buses, for example, 1:5 means that for every bus in the internal system, there are five in the external system. The computational benefits depend on the ratio. For the first iteration, the mixed approach is about 5–8 times faster for LU factorisation and 3–4 times faster for forward/back substitution than the ac model depending on the ratios of internal to external buses.

In addition, the mixed approach can be even faster with the iteration of the NR method. It does not require updating the Jacobian elements related to the dc model because the $\boldsymbol{B}$ matrix of the dc model is constant. Therefore computational benefits can be achieved by storing the elements obtained from the first iteration. The components of Jacobian matrix required to be updated are only for the ac model. When it is assumed that the NR method converges at the fourth iteration, the total number of operations with the mixed approach is 10–30 times smaller for LU factorisation and 6–15 times smaller for forward/back substitution than with the ac model depending on the ratios of internal to external buses.

4 Case study

Case studies were performed with the IEEE 118-bus system [24]. For the purpose of investigation, three simulations were conducted by changing the internal system as shown in Fig. 4. If the internal system for Test 1 is chosen for

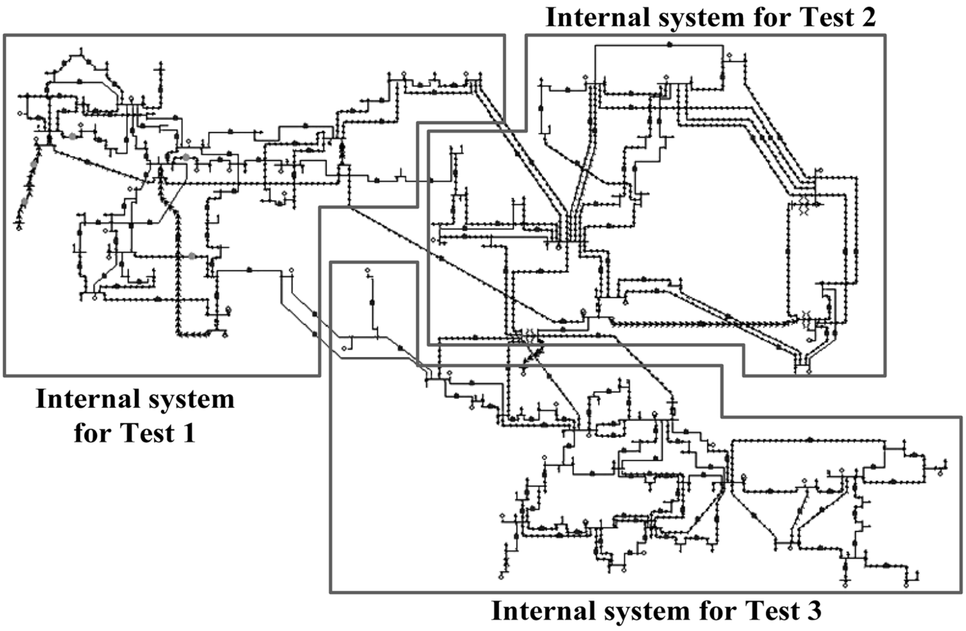

Fig. 4 *Three internal systems selected for simulation with the IEEE 118-bus system*

Table 2 Details of the division for three simulations

	Internal system buses	Boundary buses
test 1	1–43, 113, 114, 115, 117 (47 buses)	44, 49, 65, 70, 72
test 2	44–68, 116 (26 buses)	38, 42, 43, 81
test 3	70–112, 118 (44 buses)	24, 68

simulation, then everything else would be either the external system or the boundary buses. Details of this division of the IEEE 118-bus system for three simulations are also provided in Table 2.

Errors are evaluated by comparing the states using the ac model in the whole system with those using the mixed approach. In addition, errors from the dc model are also computed to compare it with the proposed approach. A Euclidean norm of the errors (10) and a sum of the Euclidean norm of the errors (11) in the internal system are commonly used for state estimation [25]. They combine both voltage-magnitude and voltage phase-angle errors.

$$\text{EN}_i = \|\overline{V_{i(\text{AC})}} - \overline{V_{i(\text{estimate})}}\| \tag{10}$$

$$\text{Sum of EN} = \sum_{i \in \text{internal}} \|\overline{V_{i(\text{AC})}} - \overline{V_{i(\text{estimate})}}\| \tag{11}$$

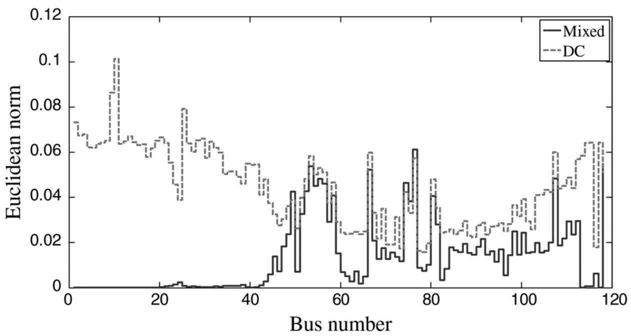

Fig. 5 *Test 1 simulation results (internal system: from buses 1 to 43 and 113, 114, 115, 117)*

Simulation results are provided in Figs. 5–7 and Table 3. Figs. 5–7 show the Euclidean norm of the error at each bus. The error in the selected internal system is small and it means that the states obtained from the mixed approach in the internal system are reasonably close to those using the ac model.

In Table 3, the sum of the Euclidean norm of the all bus errors in the internal system is calculated. The results from the mixed approach are compared to those from the dc

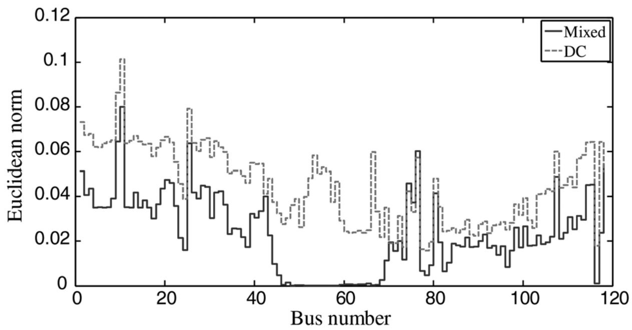

Fig. 6 *Test 2 simulation results (internal system: from buses 44 to 68 and 116)*

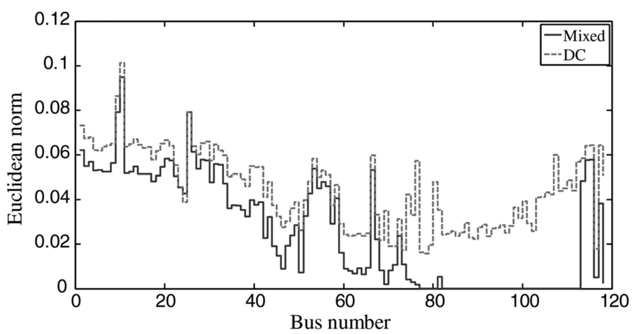

Fig. 7 *Test 3 simulation results (internal system: from buses 70 to 112 and 118)*

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Table 3 Sum of the Euclidean norm from three simulations

	Mixed approach (*A*)	DC model (*B*)	Error rate of mixed approach compared to dc model (=*A*/*B* × 100%)
test 1	0.0236	2.8853	0.82%
test 2	0.0269	0.9215	2.92%
test 3	0.0713	1.4502	4.92%

Table 4 Computation time of three case studies

	AC model	Mixed approach		
		Test 1	Test 2	Test 3
ratio of internal to external buses	–	1:1.4	1:3.3	1:1.6
computation time, s	1.37	0.74	0.56	0.65
percentage of time	100%	54%	41%	47%

model. The sum of the errors in the internal system from the mixed approach is small compared to the dc model. The error value is dependent on the test cases. The boundary assumptions for the mixed approach may introduce different amounts of errors depending on the simulation cases.

Table 4 shows the computation time of three test cases and the results from the mixed approach are compared to that from the ac model. All cases are converged at the fourth iteration. The simulation time is dependent on the case study having different ratio of internal to external buses. More speed benefits can be achieved with higher ratio in practical larger power systems.

5 Conclusion

This paper explores the development of power flow algorithms for combining the detailed AC model with the less detailed dc model. The approach gives an advanced power flow model, which has a fast solution, about10–30 times for LU factorisation and 6–15 times for forward and back substitution faster than using the ac model alone, without sacrificing accuracy in areas of interest. It can be used for any size of power system. However, more benefits in terms of speed can be achieved with larger power system case and higher dimension of the external system compared to the internal one. The approach also includes the external system, in contrast to the network equivalent technique. This approach can be utilised in a variety of power system applications.

6 Acknowledgments

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