Generating Statistically Correct Random Topologies for Testing Smart Grid Communication and Control Networks

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***Abstract*—In order to design an efficient communication scheme and examine the efficiency of any networked control architecture in smart grid applications, we need to characterize statistically its information source, namely the power grid itself. Investigating the statistical properties of power grids has the immediate benefit of providing a natural simulation platform, producing a large number of power grid test cases with realistic topologies, with scalable network size, and with realistic electrical parameter settings. The second benefit is that one can start analyzing the performance of decentralized control algorithms over information networks whose topology matches that of the underlying power network and use network scientific approaches to determine analytically if these architectures would scale well. With these motivations, in this paper we study both the topological and electrical characteristics of power grid networks based on a number of synthetic and real-world power systems. The most interesting discoveries include: the power grid is sparsely connected with obvious *small-world* properties; its nodal degree distribution can be well fitted by a mixture distribution coming from the sum of a truncated geometric random variable and an irregular discrete random variable; the power grid has very distinctive graph spectral density and its algebraic connectivity scales as a power function of the network size; the line impedance has a heavy-tailed distribution, which can be captured quite accurately by a clipped double Pareto lognormal distribution. Based on the discoveries mentioned above, we propose an algorithm that generates random topology power grids featuring the same topology and electrical characteristics found from the real data.**

***Index Terms*—Graph models for networks, power grid topology.**

I. Introduction

An electrical power grid is a critical infrastructure, whose reliable, robust, and efficient operation greatly affects national economics, politics, and people's everyday life. In the United States the bulk electric power system is operating ever closer to its reliability limits. The past decade has seen many efforts to achieve a *smart grid*, that is, using digital technology to save energy, reduce cost, and increase reliability and transparency. Especially efforts have been made to seek preventive and restorative methods for dealing with likely widespread catastrophic failures, which are caused either by unanticipated disturbances, such as the North American blackout on 14 August 2003, or intentional attacks. It has been widely agreed that there are inseparable interdependencies between reliable, robust, and efficient operations of power grids and the efficient placement and operation of related telecommunication networks, as pointed out in the work of Heydt *et al.* [8].

A. Motivation of the Work

One of our research questions is *what kind of communication network is needed to support the decentralized control of a smart grid?* To answer this question we need to do what communication designers have done in designing the large public switched telephone network (PSTN), the Internet, and the cellular networks: understanding the nature of the source and of the traffic it generates.

One first step towards the goal of producing a statistical model for the data traffic is being able to generate a large number of random power grid test cases with realistic topologies, with scalable network size, and with realistic electrical parameter settings. We contend that this first step is needed in order to properly address the network control needs of power networks; because the traditional practice of using a small number of historical test systems (e.g., IEEE model systems) is no longer sufficient for our research. Therefore, it is essential to have a statistical model for power networks both as a simulation tool to generate such power grid test cases, as well as possibly an analytical tool to grasp *what class of communication network topologies* needs to match the underlying power networks, so that the provision of the network control problem can be done efficiently.

Power grid statistics have been studied in the past by a number of researchers whose contributions are highlighted throughout the paper.

B. Previous Work and Our Contributions

Several other researchers noticed the similar needs to generate scalable-size power grid test cases. Different power grid models were proposed based on observed statistical characteristics. For example, Parashar and Thorp [11] used ring-structured power grids to study the pattern and speed of contingency or disturbance propagation. Carreras *et al.* [3] used a tree-structured power grid model to study power grid robustness and to

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detect critical points and transitions in the transmission network to cause cascading failure blackouts. Both the models provided some perspectives to power grid characteristics. But the topology of the power grids generated, i.e., ringlike or treelike structures, does not correctly or fully reflect that of a realistic power system.

The work by Watts and Strogatz [20] in the context of their work on random graphs first proposed statistically modeling the power grid as a *small-world* network. The topology of a *small-world* network is distinguished by a much shorter average path length (in hops) and a much higher clustering coefficient, compared to that of an Erdös–Rényi rand-graph network with the same network size and the same number of links.

What our work adds to the literature is a comprehensive study of all the topological as well as electrical features of the power grid network, which clarifies and fills gaps left by previous works on the subject.

Although the power grid topology manifests obvious *small-world* properties, it is in fact very sparsely connected with a very low average nodal degree $\langle k \rangle = 2 \sim 5$, which does not scale with the network size. Therefore, the *small-world* model in [20] proposed by Watts and Strogatz, which requires $\langle k \rangle \ll N \ll e^{\langle k \rangle}$ for network connectivity, in fact has a scaling property that cannot be validated by power grid topologies. Further study on the network connectivity shows that the connectivity of power grids has a very special scaling property versus the network size, which is much better than what comparably sparse Watts–Strogatz *small-world* models typically attain.

Most other studies on the power grid topology, like [2], [4], and [16], concluded that the node degree in a power grid follows an exponential (or equivalently, a geometric) distribution because its empirical probability mass function (PMF) contains an obvious exponential component. However, our study has found that for the range of small node degrees (such as $k \leq 3$), the empirical PMF curve clearly deviates from that of a geometric distribution. As a matter of fact, this phenomenon is observable in many available power grid data set that describe the topology of real-world power grids, both for the United States [2] [12] and for the European power grids [15], [16]. As we shown in [19], this deviation from a pure geometric distribution substantially affects the topology vulnerability of a network under intentional attacks. In this work, we use the probability generation function (PGF) to analyze the node degree distribution in power grids and find that it can be well fitted by a mixture distribution coming from the sum of a truncated geometric random variable and an irregular discrete random variable. We also propose a method to estimate the distribution parameters by analyzing the poles and zeros of the average PGF.

We also study the distribution of line impedances $Z_{\rm pr}$ in the power grid. Because the line impedances, together with the network topology, dominate the coupling of network components, and the propagation of oscillations in the grid. However, to our best knowledge, the analysis on the line impedance distribution, except for our efforts, has never been employed before. Our study shows that the line impedance in the power grid is heavy-tailed and can be captured quite accurately by a clipped double Pareto lognormal (DPLN) distribution.

We utilize our findings mentioned above to provide a simulation platform that generates realistic power grids with realistic topologies, with scalable network size, and with realistic electrical parameter settings, and that can aid in the evaluation of networked control protocols.[1]

The rest of the paper is organized as follows. Section II discusses system model for power grid network. Sections III and IV present our study results on the topological and electrical characteristics of realistic power grids. Section V describes in details the model of RT-*nested-Smallworld*. Finally, Section VI concludes the paper.

II. System Model

The power network dynamics are controlled by its network admittance matrix Y and by power generation distribution and load settings. The generation and load settings can take relatively independent probabilistic models, and have been discussed in the paper by Wang *et al.* [17]. Here in this work we mainly focus on the network admittance matrix, which is defined as

$$Y = A^T \Lambda^{-1}(Z_{\rm pr}) A \tag{1}$$

where $\Lambda^{-1}(\cdot)$ means taking the inverse of a diagonal matrix from a vector. The network admittance matrix has two components: the line impedance vector $Z_{\rm pr}$ and the line-node incidence matrix A. If the network has N nodes, m links, its line-node incidence matrix A, with the size of $m \times N$, can be written as

$$A : \begin{cases} A(t,i) = 1 \\ A(t,j) = -1 \\ A(t,k) = 0, \quad \text{with } k \neq i \text{ or } j \end{cases} \tag{2}$$

if the t-th link is from node i to node j.

The Laplacian matrix L can be obtained as $L = A^T A$ with

$$L(i,j) = \begin{cases} -1, & \text{if there exists link } i-j, \text{ for } j \neq i \\ k, & \text{with } k = -\sum_{j \neq i} L(i,j), \text{ for } j = i \\ 0, & \text{otherewise,} \end{cases} \tag{3}$$

with $i, j = 1, 2, \ldots, N$.

III. Topological Characteristics of Power Grids

A number of researchers have studied the statistical properties of the topology of an electrical grid, e.g., Watts and Strogatz [20], Newman [10], Whitney and Alderson [21], *et al.* The metrics they studied include some basic ones, such as network size N, the total number of links m, average nodal degree $\langle k \rangle$, average shortest path length in hops $\langle l \rangle$, etc., and more complex ones, such as the ratio of nodes with larger nodal degrees than $\overline{k}$, $r\{k > \overline{k}\}$, and the Pearson coefficient (introduced by Francis Galton in the 1880s, and named after Karl Pearson; see Rodgers and Nicewander [14]). All the metrics studied by previous researchers can be derived from the graph Laplacian. For example, the total number of links is

[1]Only a small portion of the above results were shown or mentioned in one of our conference papers [18] and due to the strict paper length limitation of the conference, many details had to be trimmed even for the mentioned parts.

TABLE I
TOPOLOGICAL CHARACTERISTICS OF REAL-WORLD POWER NETWORKS

	(N,m)	$\langle l \rangle$	$\langle k \rangle$	ρ	$r\{k > \overline{k}\}$
IEEE-30	(30,41)	3.31	2.73	-0.0868	0.2333
IEEE-57	(57,78)	4.95	2.74	0.2432	0.2105
IEEE-118	(118,179)	6.31	3.03	-0.1526	0.3051
IEEE-300	(300, 409)	9.94	2.73	-0.2206	0.2367
NYISO	(2935,6567)	16.43	4.47	0.4593	0.1428
WSCC	(4941, 6594)	18.70	2.67	0.0035	0.2022

$$m = \frac{1}{2}\sum_i L(i,i). \tag{4}$$

The average nodal degree is

$$\langle k \rangle = \frac{1}{N}\sum_i L(i,i). \tag{5}$$

The network adjacency matrix is

$$M_{\text{adj}} = -L + \Lambda\left(\text{diag}(L)\right). \tag{6}$$

With the adjacency matrix M_{adj} one can easily run Dijkastra's algorithm [5] to calculate average shortest path length in hops $\langle l \rangle$. The nodal degree vector is

$$\underline{k} = [k_1, k_2, \ldots, k_N] = \text{diag}(L). \tag{7}$$

And if we define $\overline{k}$ as the average degree of a node seen at the end of a randomly selected link (i,j), i.e.,

$$\overline{k} = (2m)^{-1}\sum_{(i,j)}(k_i + k_j) = (2m)^{-1}\sum_i k_i^2 = \frac{\langle k^2 \rangle}{\langle k \rangle} \tag{8}$$

then the ratio $r\{k < \overline{k}\}$ can be obtained as

$$r\{k > \overline{k}\} = \frac{\left\|\{k_i; k_i > \overline{k}\}\right\|_\infty}{N}. \tag{9}$$

The Pearson node degree correlation coefficient is a measure to evaluate the correlation of node degrees in the network

$$\rho = \frac{\sum_{(i,j)}(k_i - \overline{k})(k_j - \overline{k})}{\sqrt{\sum_{(i,j)}(k_i - \overline{k})^2(k_j - \overline{k})^2}}. \tag{10}$$

Newman [10] observed that the Pearson coefficient for some kinds of networks is consistently positive while for other kinds it is negative. Therefore, Whitney and Alderson [21] proposed to use the Pearson coefficient to differentiate technological networks from social networks. However, [21] has found that the Pearson coefficient of power grids does not have restrictive characteristics but ranges over a wide interval from negative and positive. Our evaluation experiments in Section III-A also verified this finding (see Table I).

A. Topological Characteristics

Table I shows the parameter values resulting from the IEEE model systems and the NYISO and WSCC systems based on selected metrics. For the reader's reference, the IEEE 30, 57, and 118 bus systems represent different parts of the American electric power system in the midwestern United States; the IEEE 300 bus system is synthesized from the New England power system. More information can be obtained from Washington University's "Power systems test case archive" Web site.[2] WSCC is the electrical power grid of the western United States and NYISO represents New York state bulk electricity grid.

Two discoveries pertaining to the topology of the grid are most interesting.

1) Power grids are sparsely connected. The average nodal degree does not scale as the network size increases. Instead, it falls into a very restricted range, and it is a function of the particular area where the network belongs to rather than the network size. For example, the power grids in the western United States have an average nodal degree lying somewhere between 2.5 and 3, while the grids in the northeastern United States are a little denser, with $\langle k \rangle$ around 4.5. All of them exhibit exponential tails in the node degree distribution.
2) In Watts and Strogatz's work [20] the authors hypothesized that power grids have the salient features of *small-world graphs*. That is, while the vast majority of links are similar to that of a regular lattice, with limited near neighbor connectivity, a few links connect across the network. These bridging links significantly shorten the path length that connects every two nodes and critically increase the connectivity of the network. At the same time, their scarcity puts the connectivity at risk in the case of link failure for one of these critical bridges. While the fundamental intuition is correct, we observe that small-world graphs are only partially able to capture features of the power grid. While being similarly sparse, power grids have better connectivity scaling laws than small-world graphs.

Another characterizing measure to distinguish a *small-world* network is called *clustering coefficient*, which assesses the degree to which nodes tend to cluster together. A *small-world* network usually has a clustering coefficient significantly higher than that of a random graph network, given the comparable network size and total number of edges. The random graph network mentioned here refers to the network model defined by Erdös and Rényi [6], with N labeled nodes connected by m edges which are chosen uniformly randomly from the $N(N-1)/2$ possible edges.

The clustering coefficient is defined by Watts and Strogatz as the average of the clustering coefficient for each node [20]

[2]http://www.ee.washington.edu/research/pstca/

TABLE II
THE CLUSTERING COEFFICIENTS OF REAL-WORLD POWER NETWORKS AND RANDOM GRAPH NETWORKS

	$C(G)$	$C(R)$
IEEE-30	0.2348	0.094253
IEEE-57	0.1222	0.048872
IEEE-118	0.1651	0.025931
IEEE-300	0.0856	0.009119
NYISO-2935	0.2134	0.001525
WSCC-4941	0.0801	0.000540

$$C = \frac{1}{N}\sum_{i=1}^{N} C_i \tag{11}$$

with C_i being the node clustering coefficient as

$$C_i = \frac{\lambda_G(i)}{\tau_G(i)} \tag{12}$$

where $\lambda_G(i)$ is the number of edges between the neighbors of node i and $\tau_G(i)$ the total number of edges that could possibly exist among the neighbors of node i. For undirected graphs, obviously $\tau_G(i) = k_i(k_i-1)/2$ given k_i as the node degree. As pointed out in [1], the clustering coefficient for a random graph network theoretically equals the probability of randomly selecting links from all possible links. That is, $C(R) = 2m/(N(N-1)) = \langle k\rangle/(N-1)$.

Table II shows the clustering coefficients of the IEEE power systems, the NYISO system, and the WSCC system compared to random graph networks with the same network size and same total number of links. The former is denoted as $C(G)$, and the latter as $C(R)$. The relatively large clustering coefficient of power networks was used in [20] as an indicator of the similarity between power grids and *small-world* networks. But, as we will discuss next, the topology of power grids are far more connected at large scales than *small-world* graphs produced by the proposed model in [20].

B. Nodal Degree Distribution

Sole *et al.* [16] studied the robustness of European power grids under intentional attacks and concluded that European power grids are sparsely connected with global average nodal degree of $\langle k\rangle = 2.8$ and the link distribution (i.e., nodal degree distribution) is exponential: the probability of having a node linked to k other nodes is $p(k) = \exp(-k/\gamma)/\gamma$, with the constant $\gamma = \langle k\rangle$.

In this work we examined the empirical distribution of nodal degrees $\underline{k} = \mathrm{diag}(L)$ in the available real-world power grids. Fig. 1 shows the histogram PMF in log-scale for the nodal degrees of NYISO system. If the PMF curve approximates a straight line in the semilogarithm plot (i.e., shown as $\log(p(k))$ versus k), it implies an exponential tail which is analogous to that of the geometric distribution. However, it is also noticed

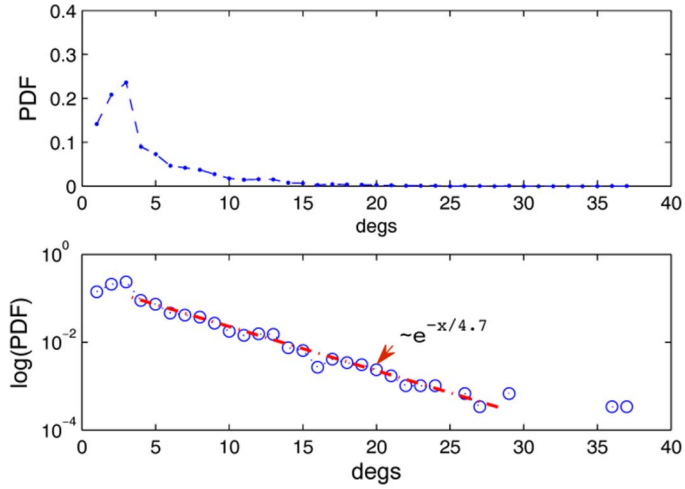


Fig. 1. Empirical pdf of nodal degrees in real-world power grids (NYISO).

that for the range of small node degrees, that is, when $k \le 3$, the empirical PMF curve clearly deviates from that of a geometric distribution.

We used the PGF to analyze the node degree distribution in power grids. The PGF of a random variable X is defined as $G_X(z) = \sum_k \Pr_{(x=k)} z^k$. Given a sample data set of X with the size of N, its PGF can also be estimated from the mean of z^X because

$$\begin{aligned} E(z^X) &= \frac{1}{N}\sum_x z^x \\ &= \sum_k \frac{n_{(x=k)}}{N} z^k \\ &\approx \sum_k \Pr{}_{(x=k)} z^k \end{aligned} \tag{13}$$

where $n_{(x=k)}$ denotes the total number of the data items equaling to k. Due to $\lim_{N\to\infty}(n_{(x=k)}/N) = \Pr_{(x=k)}$, we can have $E(z^X) \approx G_X(z)$ with a large enough data size.

If a random variable can be expressed as a sum of two independent random variables, its PMF is then the convolution of the PMFs of the components variables, and its PGF is the product of that of the component variables. That is,

$$G_X(z) = G_{X_1}(z)G_{X_2}(z). \tag{14}$$

The examination of PGFs concluded that the node degree distribution in power grids can be very well approximated by a sum of two independent random variables, that is,

$$\mathcal{K} = \mathcal{G} + \mathcal{D} \tag{15}$$

where $\mathcal{G}$ is a truncated geometric with the threshold of $k_{\max}$

$$\begin{aligned} \Pr{}_{(\mathcal{G}=k)} &= \frac{(1-p)^k p}{\sum_{i=0}^{k_{\max}} (1-p)^i p} \\ &= \frac{(1-p)^k p}{1-(1-p)^{k_{\max}+1}}, \quad k = 0,1,2,\ldots,k_{\max} \end{aligned} \tag{16}$$

with the PGF as

$$G_{\mathcal{G}}(z) = \frac{\sum_{k=0}^{k_{\max}} (1-p)^k p z^k}{1-(1-p)^{k_{\max}+1}} = \frac{p\left(1-((1-p)z)^{k_{\max}+1}\right)}{(1-(1-p)^{k_{\max}+1})(1-(1-p)z)}. \quad (17)$$

And $\mathcal{D}$ is an irregular discrete $\{p_1, p_2, \ldots, p_{k_t}\}$,

$$\Pr_{(\mathcal{D}=k)} = p_k, \quad k = 1, 2, \ldots, k_t \quad (18)$$

with the PGF as

$$G_{\mathcal{D}}(z) = p_1 z + p_2 z^2 + p_3 z^3 + \cdots + p_{k_t} z^{k_t}. \quad (19)$$

Therefore, the PMF of $\mathcal{K}$ is

$$\Pr_{(\mathcal{K}=k)} = \Pr_{(\mathcal{G}=k)} \otimes \Pr_{(\mathcal{D}=k)}. \quad (20)$$

And the PGF of $\mathcal{K}$ can be written as

$$G_{\mathcal{K}}(z) = \frac{p\left(1-((1-p)z)^{k_{\max}+1}\right)\sum_{i=1}^{k_t} p_i z^i}{(1-(1-p)^{k_{\max}+1})(1-(1-p)z)}. \quad (21)$$

Equation (21) indicates that the PGF $G_{\mathcal{K}}(z)$ has $k_{\max}$ zeros evenly distributed around a circle of radius of $1/(1-p)$ which are introduced by the truncation of the geometric $\mathcal{G}$ (because the zero at $1/(1-p)$ has been neutralized by the denominator $(1-(1-p)z)$ and has k_t zeros introduced by the irregular discrete $\mathcal{D}$ with $\{p_1, p_2, \ldots, p_{k_t}\}$.

Fig. 2 shows the contour plots of PGF of node degrees for different groups of buses in the NYISO system. Three interesting discoveries have been made: a) clearly each plot contains evenly distributed zeros around a circle, which indicate a truncated geometric component; b) besides the zeros around the circle, most contour plots also have a small number of off-circle zeros, which come from an embedding irregular discrete component; and c) the contour plot for each group of nodes has zeros with similar pattern but different positions. This implies that each group of node degrees has similar distribution functions but with different coefficients. Therefore, it is necessary and reasonable to characterize the node degrees distribution according to the node types. Otherwise if the node degrees aggregate into one single group, just as in Fig. 2(a), some important characteristics of a subgroup of node degrees would be concealed (e.g., comparing (a) and (b)–(e) in Fig. 2).

From the contour plots one can easily locate the zeros in PGF, and further determine the coefficients of corresponding distribution functions. The estimated coefficients for each group of nodes in the NYISO and all the nodes in the WSCC systems are listed in Table III. Because the WSCC system data we have only contains unweighted raw data without distinguishing node types, its node degree distribution is only analyzed for one aggregate group. Fig. 3 compares the PMF with estimate coefficients and the empirical PMF of the NYISO system and shows that the former matches the latter with quite good approximation. Fig. 4 presents the PGF contour plot and the PMF comparing plots for all the nodes in the WSCC system. The results from both systems have validated our assumption of node degree distribution in power grids: it can be expressed as a sum of a truncated geometric random variable and an irregular discrete random variable. And the results also demonstrated the effectiveness of the proposed method of analyzing node degree distribution by using the PGF.

C. Wiring the Network

The *small-world* model, proposed by Watts and Strogatz [20], is generated starting from a regular ring lattice, then using a small probability, by rewiring some local links to an arbitrary node chosen uniformly at random in the entire network (to make it a *small-world* topology). A tool to visually highlight *small-world* topologies is the Kirk graph, which was proposed by Kirk [9]. It is a simple but effective way to show a network topology: first, node numbers are assigned according to physical nodal adjacency, that is, physically closely located nodes are given close numbers; then all the nodes are sequentially and evenly spread around a circle and links between nodes are drawn as straight lines inside the circle. Fig. 5 shows three representative network topologies, using Kirk graphs, of an Erdös–Rényi Rand-graph network, of a Watts–Strogatz small-world network, and of a realistic power grid—IEEE-57 network. The three networks have same network size and almost same total number of links.

One can easily notice the topology difference between the Erdös–Rényi network and IEEE-57 network while observing as well a good degree of similarity between the Watts and Strogatz *small-world* model and the test power network.

The figure also shows that the rewiring of the test power grid is not independent, as in that of Watts and Strogatz *small-world* model; instead, long hauls appear over clusters of nodes. One property of the IEEE 57 network that is visually noticeable, and that differentiates it from the *small-world* graph, is the fact that the nodes that are rewired appear to be in clusters, rather than being chosen independently. This fact has an intuitive explanation: long hauls require having a right of way to deploy a long connection and it is highly likely that the long wires will reuse part of this space. These physical and economical constraints inevitably affect the structure of the topology.

However, there is more than what meets the eye. In our study we found that the small-world model proposed by Watts and Strogatz has scaling property that cannot be validated by power grid topologies, precisely because the average nodal degree of a power network is almost invariant to the size of the network. Given a network size with its specified average nodal degree, the model cannot produce a connected power grid topologies for reasonably large network sizes using realistic power grid degree distributions. The main reason for the poor scaling lies in the

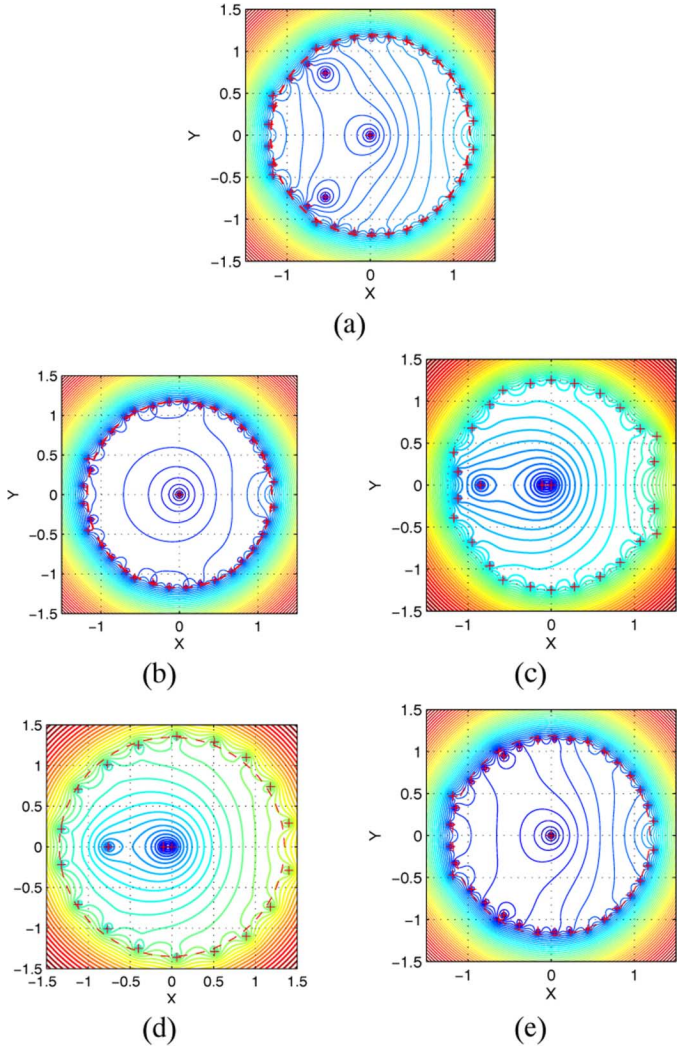


Fig. 2. The contour plot of $E(z^X)$ of node degrees for different groups of buses in the NYISO system. (a) All buses. (b) Gen buses. (c) Load buses. (d) Connection buses. (e) Gen+Load buses. The zeros are marked by red $+$'s.

TABLE III
ESTIMATE COEFFICIENTS OF THE TRUNCATED GEOMETRIC AND THE IRREGULAR DISCRETE FOR THE NODE DEGREES IN THE NYISO AND WSCC SYSTEM

node groups	max($\underline{k}$)	p	k_{max}	k_t	$\{p_1, p_2, \cdots, p_{k_t}\}$
All	37	0.2269	34	3	0.4875, 0.2700, 0.2425
Gen	37	0.1863	36	1	1.000
Load	29	0.2423	26	3	0.0455, 0.4675, 0.4870
Conn	21	0.4006	18	3	0.0393, 0.4442, 0.5165
Gen+Load	37	0.2227	34	3	0.4645, 0.3385, 0.1970
All-WSCC	19	0.4084	16	3	0.3545, 0.4499, 0.1956

fact that in order to produce a sparse but connected topology, the Watts and Strogatz *small-world* model requires [1], [20]

$$1 \ll \ln(N) \ll \langle k \rangle \ll N \tag{22}$$

or equivalently as

$$\langle k \rangle \ll N \ll e^{\langle k \rangle}. \tag{23}$$

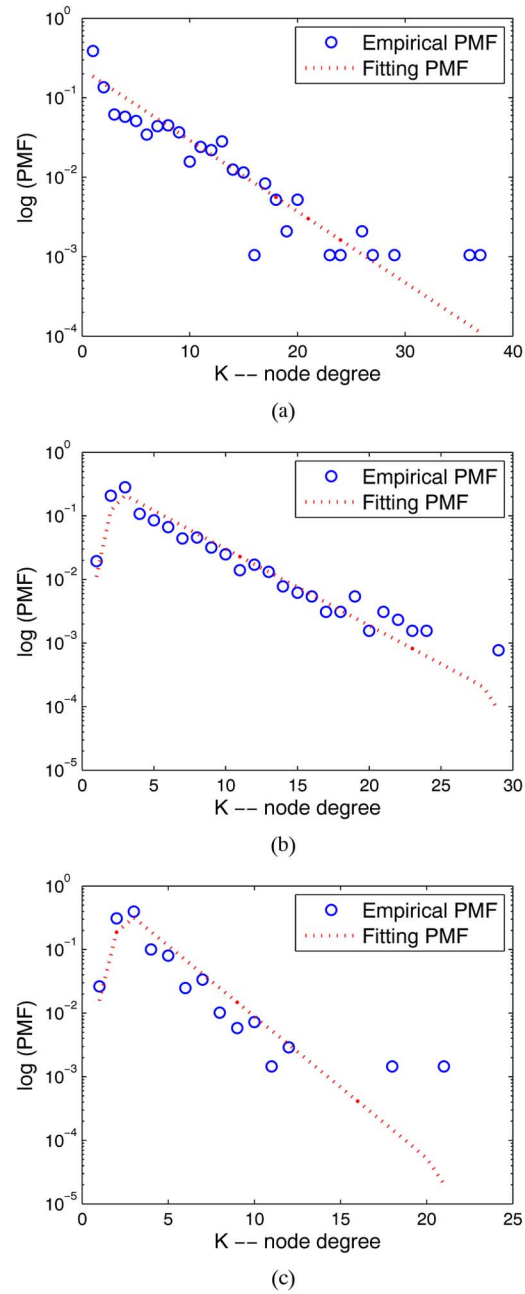
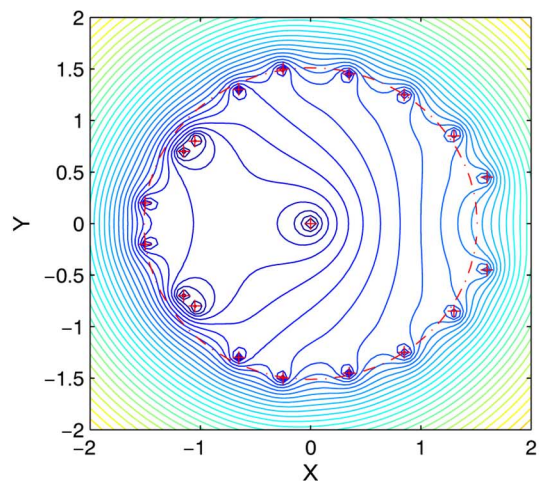


Fig. 3. Comparing the empirical and fitting PMF of node degrees in NYISO. (a) Gen buses. (b) Load buses. (c) Connection buses.

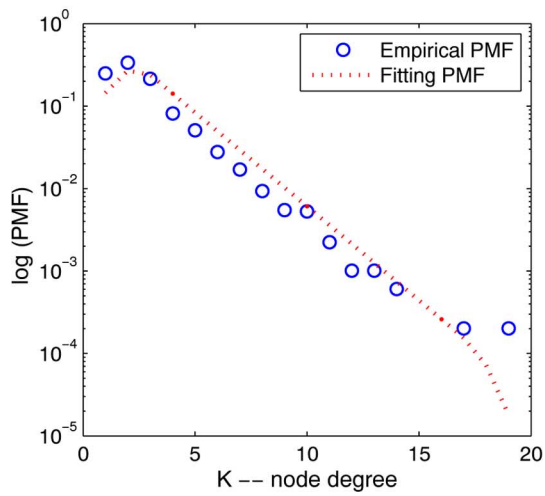
On the other side, real-world power grids have a very low average nodal degree with $\langle k \rangle = 2 \sim 3$ or $4 \sim 5$, regardless of the network size. This will limit the network size much smaller than 20 or 150 in order to produce a connected topology using the model. However, in the real world, large power grids are connected even if they are much sparser than what is required by Watts–Strogatz *small-world* model.

D. Graph Spectrum and Connectivity Scaling Property

In this work we also examined the eigenvalues of the adjacency matrix M_{adj} and the Laplacian matrix L for power grid networks because the eigenvalues carry important topological features of a network. The two matrices are exchangeable, as



(a)



(b)

Fig. 4. Distribution of node degrees in WSCC—all buses. (a) Contour plot of PGF. (b) Comparing PMFs.

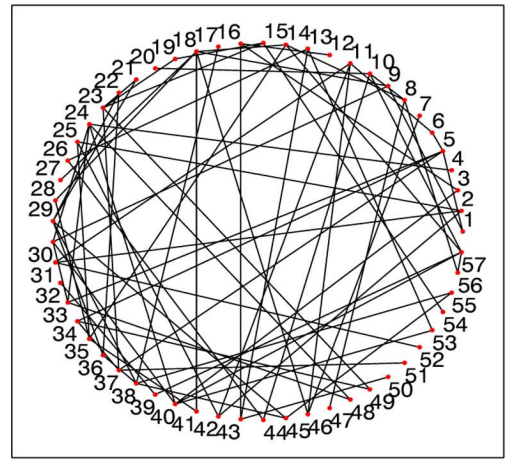
shown in Section III-A and either of them fully describes a network topology.

The set of eigenvalues of its adjacency matrix M_{adj} is called the spectrum of a graph. *Graph spectral density* is defined as

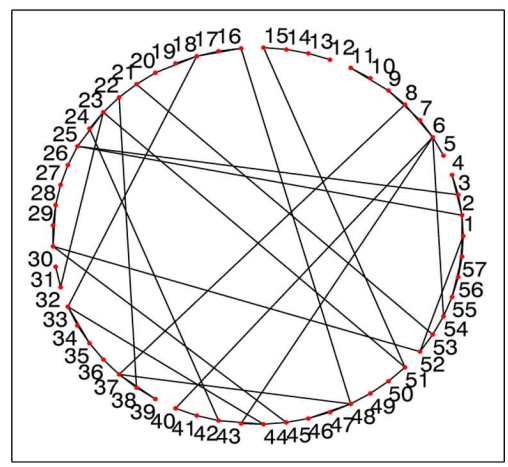
$$\rho(\lambda) = \frac{1}{N}\sum_{j=1}^{N}\delta(\lambda - \lambda_j) \tag{24}$$

with $\{\lambda_j, j = 1, 2, \ldots, N\}$ forming the graph spectrum of the network. The work of Albert and Barabási [1] pointed out that the kth moments of $\rho(\lambda)$ represents the average number of k-hop paths returning to the same node in the graph. However, please note that these paths can contain nodes which are already visited.

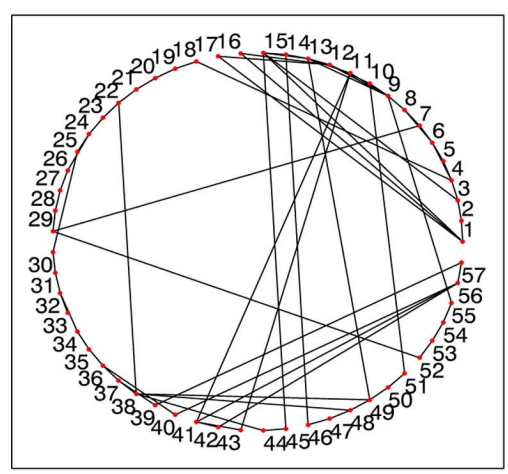
It is stated in [1] that the *graph spectral density* of network topology from different categories has distinctively different patterns. A Erdös–Rényi random graph network $G(N;p)$, given a large network size N and a nontrivial link selection probability $p(N) = cN^{-z}$, with $z < 1$ and c as a constant regardless of network size, has its normalized spectral density converging to a semicircular distribution as $N \to \infty$. This is known as Wigner's semicircle law [22], [23]. That is,



(a)



(b)



(c)

Fig. 5. Topology comparison using Kirk plot. (a) Erdös–Rényi Rand-graph network. (b) Watts–Stogatz small-world. (c) IEEE 57.

$$\rho(\lambda) = \begin{cases} \dfrac{\sqrt{4-\left(\frac{\lambda}{\lambda_0}\right)^2}}{2\pi\lambda_0} & \text{if } |\lambda| < 2\lambda_0 \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

with $\lambda_0 = \sqrt{Np(1-p)}$ and $p = \frac{m}{N(N-1)/2} = \frac{\langle k\rangle}{N-1}$, where m is the total number of links and $\langle k\rangle$ is the average node degree.

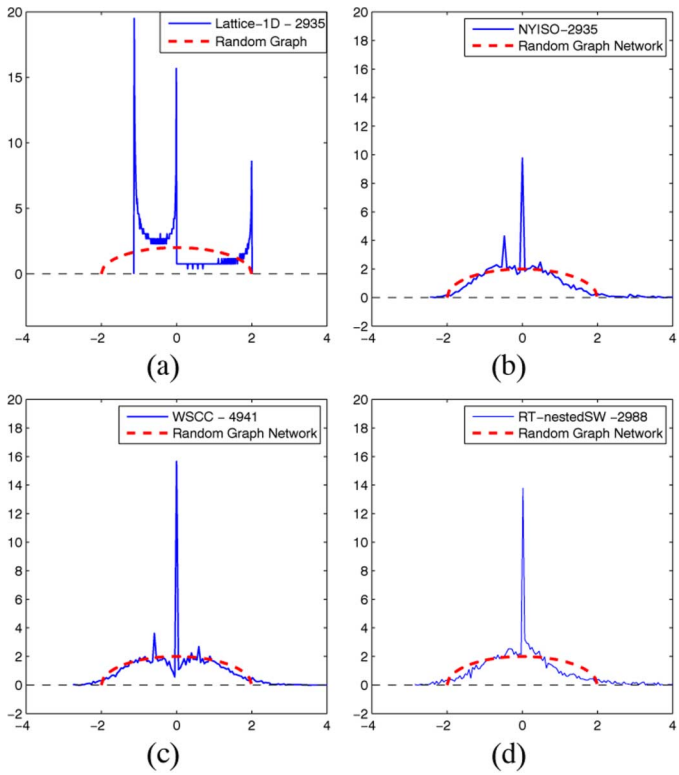


Fig. 6. The normalized graph spectral density of different networks, $\widetilde{\rho}(\lambda)$ versus $\widetilde{\lambda}$: the dotted line of the semicircle represents the graph spectral density of random graph networks. (a) A ring lattice. (b) NYISO. (c) WSCC. (d) RT-*nested-Smallworld*.

And $\widetilde{\rho} = 2\pi\lambda_0\rho(\lambda) = \sqrt{4-\widetilde{\lambda}^2}$ with $\widetilde{\lambda} = \lambda/\lambda_0$ depicts a semicircle around the origin with a radius of 2.

Fig. 6 shows the normalized spectral density of a ring lattice [Fig. 6(a)], of two real-world power grids [Fig. 6(b) and (c)], and of a power grid generated from our proposed model RT-*nested-Smallworld* [Fig. 6(d)], compared to the semicircle law corresponding to random graphs with the same network size and number of links. The plots demonstrate that power grid has very distinctive spectral density rather than that of lattice network or random graph network; and that the proposed model RT-*nested-Smallworld* produces a very good match to the real data in terms of graph spectral density.

Another important measure is the second smallest eigenvalue of the Laplacian matrix, $\lambda_2(L)$, called the algebraic connectivity. This measure is sometimes termed as Fiedler eigenvalue, due to the fact that it was first introduced by Fiedler [7]. $\lambda_2(L)$ reflects how well a network is connected and how fast information data can be shared across the network. As a fact the smallest eigenvalue of the Laplacian is always zero, i.e., $\lambda_1(L) \equiv 0$ and the number of times 0 appears as an eigenvalue in the Laplacian is the total number of connected components in the network. The eigenvalue $\lambda_2(L)$ is greater than 0 if and only if network is a connected graph. If the algebraic connectivity $\lambda_2(L)$ is close to 0, the network is close to being disconnected. Otherwise, if $\lambda_2(L)/n$ tends to be 1, with n as the network size, the network tends to be fully connected.

Table IV shows the algebraic connectivity of IEEE model systems and the NYISO system. Fig. 7 plots the connectivity

TABLE IV
ALGEBRAIC CONNECTIVITY OF REAL-WORLD POWER NETWORKS

	$\lambda_2(L)$
IEEE-30	0.21213
IEEE-57	0.088223
IEEE-118	0.027132
IEEE-300	0.0093838
NYISO-2935	0.0014215
WSCC-4941	0.00075921

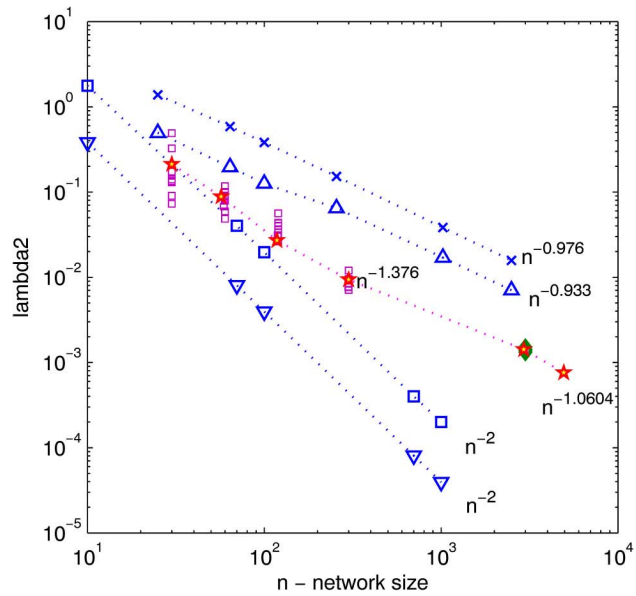


Fig. 7. Connectivity scaling curve versus network size: 2-D lattice with $k = 4$ (blue x-dotted line); 2-D lattice with $k = 2$ (blue $\triangle$-dotted line); 1-D lattice with $k = 4$ (blue $\square$-dotted line); 1-D lattice with $k = 2$ (blue ∇-dotted line); nested-Smallworld RT with subnetwork-size $= 30$ (small dark red $\square$); nested-Smallworld RT with subnetwork-size $= 300$ (green $\diamond$); power grids (red star).

scaling curve of power grid versus network size and compares it with that of 1-D and 2-D lattices. One-dimensional lattice is a ring-structured topology, with nodes connected with most adjacent neighbors on both sides. Two-dimensional lattice is a regular 2-D meshed grid with each boundary side merging with the other side and each node connected to the most adjacent neighbors around it. For 1-D lattice, its connectivity scales as $\lambda_2(L) \propto n^{-2}$; for 2-D lattice, its connectivity grows as $\lambda_2(L) \propto n^{-1}$; interestingly, for power grids, its connectivity grows as $\lambda_2(L) \propto n^{-1.376} \sim n^{-1.0604}$, lying between those of 1-D lattice and 2-D lattice.

In trying to fit the random wiring of power network we postulated a new possible model, RT-*nested-Smallworld*. The model resulted from nesting several *small-world* subnetworks, whose size is likely to produce a connected topology with a realistic degree distribution, into a regular lattice again as shown in Fig. 8. This model is presented with more details in Section V. The intuition guiding our modeling was that the network would have produced connected topologies with a connectivity that was intermediate between the 1- and 2-D lattices. As shown in Fig. 7, there is an excellent match between the algebraic connectivity of this type of model and the real data.

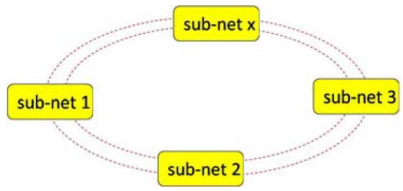


Fig. 8. Lattice-connections between subnetworks.

IV. Electrical Characteristics

Another observation that guided our work and distinguishes our contribution from the referenced ones above is as follows: rather than focusing only on the network topology of power grid, the efforts we have been made were aimed at reproducing accurately its electrical characteristics as well. One key element for a power grid network is its network admittance matrix Y, which can be expressed as a function of network adjacency matrix and line impedances. The topology and electrical characteristics of any power grid can be deduced from or closely related with them. Therefore, our proposed power grid models include components which assign transmission line impedances according to specific distributions.[3] We proposed in our previous work by Wang *et al.*[17], two models referred to as RT-*Uniform* and RT-*Poisson*, because they were generated starting from a uniform and a Poisson distribution for the nodal location respectively (emulating what is often done in modeling wireless networks).We are not aware of other research efforts to model the line impedances.

In our previous work we evaluated and compared their performance based on topological metrics as well as electrical characteristics to those of the test networks. The main issues with these models were their inability to reproduce the empirical IEEE power system line impedance distribution as the likely one based on the model. Incidentally, the models proposed in [17], also had higher connectivity than the IEEE model power systems.

The data on line impedances of power grids we used are again from IEEE model systems and the NYISO system. The first clear observation from the empirical histogram probability density distribution (pdf) is that the distribution of the line impedances is heavy tailed. In searching among the heavy tailed distributions for a fit, we estimated via the maximum likelihood (ML) criterion the parameters of the candidate distribution from the data and used the appropriately modified Kolmogorov–Smirnov test (K-S test) to check if the hypothesized distribution was a good fit. The candidate distribution functions include gamma, generalized Pareto (GP), lognormal, double Pareto lognormal (DPLN), and two new distributions that we call lognormal-clip and DPLN-clip, which will be explained later.

The DPLN distribution was introduced by Reed and Jorgensen [13], which proved to be very useful to model the size distributions of various phenomena, like incomes and earnings, human settlement sizes, etc. We applied the DPLN distribution to fit the line impedances of power grids because the latter implicitly relate to human settlement size in the network. Generally speaking, large generation is located near water or the fuel source, but remotely away from densely populated area with large scale human settlements. Therefore, long-distance transmission lines have been constructed in order to provide the interconnection and serve the large scale settlements. And the long-distance lines usually exhibit large line impedance.

The lognormal-clip and DPLN-clip are especially suited for fitting the NYISO data because the line impedances in this system appear to approximate a lognormal or DPLN distribution very well except for having an *interrupted* tail, which is captured by the *clipping*. Therefore, we assume the NYISO data is resulted from some original impedance data being *clipped* by an exponential cutoff tailing coefficient. That is, given the original impedance data Y following a specific distribution, $Y \sim f_Y(y)$, the clipped impedance data is

$$X = Z_{\max}\left(1 - e^{-Y/Z_{\max}}\right) \tag{26}$$

with $Z_{\max}$ being the cutoff threshold, so that

$$Y = -Z_{\max}\log\left(1 - \frac{X}{Z_{\max}}\right). \tag{27}$$

The resulting clip distribution turns out to be

$$\begin{aligned} X \sim f_X(x) &= Y'_{(x)} f_Y\left(Y_{(x)}\right) \\ &= \frac{Z_{\max}}{Z_{\max} - x} f_Y\left(-Z_{\max}\log\left(1 - \frac{x}{Z_{\max}}\right)\right). \end{aligned} \tag{28}$$

We believe that the introduction of the clipping mechanism is reasonable. This is because in real-world power grids, due to the expensive right-of-way cost plus the high construction and maintenance cost, transmission lines are limited in length and correspondingly in the line impedance, which is proportional to the length.

The distribution functions are listed below.

Gamma:

$$\Gamma(x \mid a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{x/b}. \tag{29}$$

Generalized Pareto (GP):

$$gp(x \mid k, \sigma, \theta) = \left(\frac{1}{\sigma}\right)\left(1 + k\frac{(x-\theta)}{\sigma}\right)^{-1-(1/k)}. \tag{30}$$

Lognormal:

$$\log n(x \mid \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\log x - u)^2}{2\sigma^2}}. \tag{31}$$

DPLN:

$$\begin{aligned} &\mathrm{DPLN}(x|\alpha, \beta, \mu, \sigma) \\ &= \frac{\alpha\beta}{\alpha+\beta}\Bigg[A(\alpha, \mu, \sigma)x^{-\alpha-1}\Phi\left(\frac{\log x - \mu - \alpha\sigma^2}{\sigma}\right) \\ &\qquad + A(-\beta, \mu, \sigma)x^{\beta-1}\Phi^c\left(\frac{\log x - \mu + \beta\sigma^2}{\sigma}\right)\Bigg]. \end{aligned} \tag{32}$$

where $A(\theta, \mu, \sigma) = e^{(\theta\mu + \theta^2\sigma^2/2)}$.

[3]The transmission line impedance in a power grid is in fact a complex number. When it comes to the distribution analysis, we study the distribution of the magnitude of Z_{pr}. The line impedance can be represented as $Z_{\mathrm{pr}} = R + jX$, where R is the resistance and X the reactance. Usually X is the dominant component, whereas R only takes a trivial value, which in many cases can even be neglected. Therefore, one can easily reconstruct the complex value of Z_{pr} given its magnitude.

TABLE V
DISTRIBUTION FITTING FOR LINE IMPEDANCES

System	Fitting Distribution	ML Parameter Estimates (*alph*=0.05)
IEEE-30	$\Gamma(x\|a,b)$	$a = 2.14687$ $b = 0.10191$
IEEE-118	$\Gamma(x\|a,b)$	$a = 1.88734$ $b = 0.05856$
IEEE-57	$gp(x\|k,\sigma,\theta)$	$k = 0.33941$ $\sigma = 0.16963$ $\theta = 0.16963,$
IEEE-300	$gp(x\|k,\sigma,\theta)$	$k = 0.45019$ $\sigma = 0.07486$ $\theta = 0.00046,$
NYISO-2935	$logn_{clip}(x\|\mu,\sigma,Z_{\max})$	$\mu = -2.37419$ $\sigma = 2.08285$ $Z_{max} = 1.9977$
	$dPlN_{clip}(x\|\alpha,\beta,\mu,\sigma,Z_{\max})$	$\alpha = 44.25000$ $\beta = 44.30000$ $\mu = -2.37420$ $\sigma = 2.082600$ $Z_{max} = 1.9977$

Lognormal-clip:

$$\log n_{clip}(x \mid \mu,\sigma,Z_{\max}) = \frac{Z_{\max}}{Z_{\max}-x}\log n\left(-Z_{\max}\log\left(1-\frac{X}{Z_{\max}}\right) \mid \mu,\sigma\right). \quad (33)$$

DPLN-clip:

$$\mathrm{DPLN}_{clip}(x \mid \alpha,\beta,\mu,\sigma,Z_{\max}) = \frac{Z_{\max}}{Z_{\max}-x} \times \mathrm{DPLN}\left(-Z_{\max}\log\left(1-\frac{X}{Z_{\max}}\right) \mid \alpha,\beta,\mu,\sigma\right). \quad (34)$$

With the ML parameter estimates from each candidate distribution function, K-S tests are run to pick the distribution that gives the best K-S test result. Table V shows the best-fitting distribution function with the corresponding ML parameter estimates. Fig. 9 compares the empirical pdf's and CDFs (in logarithm scales) with those from best-fitting distribution function for the systems of IEEE 300 and NYSIO. Our studies show that gamma distribution is the best fit for the line impedances of IEEE 30 and 118 systems and GP distribution for that of IEEE 57 and 300 systems; while for the NYISO system, lognormal-clip and DPLN-clip are the best fits. We believe that the line impedances in real-world power grids have heavy-tailed distributions. This is also reasonable because, even if not one to one, the line impedance grows with the distance covered by the bus.

Since the distribution of human settlements tends to be clustered, it is reasonable that long tails appear between these clusters of smaller size buses, giving rise to these heavy tails.

It also seems to agree with the topological properties of the graph, and it is reasonable to consider these heavy weight impedances as good candidates for line impedances of the links that are rewired in the small-world islands, as well as the ones connecting difference islands in our *nested- Smallworld* model.

V. GENERATION OF RANDOM TEST NETWORKS

In this section we present a new random topology power grid model, RT-*nested-Smallworld*. This model constructs a large-scale power grid using a hierarchical way: first form connected subnetworks with size limited by the connectivity requirement; then connect the subnetworks through lattice connections; finally, generate the line impedances from some specific distribution and assign them to the links in the topology network. The hierarchy in the model arises from observation of real-world power grids: usually a large-scale system consists of a number of smaller-size subsystem (e.g., control zones), which are interconnected by sparse and important tie lines.

The model mainly contains three components: a) *clusterSmallWorld* subnetwork; b) *lattice* connections; and c) generation and assignment of line impedances; which will be respectively described in detail in Sections V-A–C.

A. clusterSmallWorld Subnetwork

As mentioned in the previous section, power grid topology has *small-world* characteristics; it is sparsely connected with a low average nodal degree not scaling with the network size. On the other side, in order for a *small-world* model to generate a connected topology, the network size has to be limited. In our proposed model, different mechanisms from that of Watts–Strogatz *small-world* model have been adopted to form a power grid subnetwork in order to improve its resulting connectivity, as shown in the following paragraphs. Consequently, the connectivity limitation on the network size can be expanded from what is indicated by (23). The experiments have shown that: for $\langle k\rangle = 2 \sim 3$, the network size should be limited no greater than 30; and for $\langle k\rangle = 4 \sim 5$, 300.

Therefore, the first step of this new model is to select the size of subnetworks according to connectivity limitation. Then a topology is built up through a modified *small-world* model, called *clusterSmallWorld*. This model is different from the Watts–Strogatz small-world model in two aspects: the link selection and the link rewires. That is, instead of selecting links to connect most immediate $\langle k\rangle/2$ neighbors to form a regular lattice, our model selects a number k of links at random from a local neighborhood N_{d_0} with the distance threshold of d_0, where k comes from a geometric distribution. The local neighborhood is defined as the group of close-by nodes with mutual node number difference less than the threshold d_0, that is, $N_{d_0}^{(i)} = \{j;\, |j-i| < d_0\}$ for node i. It is worth noting that our model adopts a geometric distribution with the expectation of $\langle k\rangle$ for the initial node degree settings (i.e, for the link selection). Our experiments have shown that the process of link selection and link rewiring that follows will transform the initial node degree distribution and finally result in a distribution with good matching approximation to the observed ones as shown in Section III-B.

For the link rewires, the Watts–Strogatz small-world model selects a small portion of the links to rewire to an arbitrary node chosen at random in the entire network. Our clusterSmallWorld model instead uses a Markov chain with transition probabilities

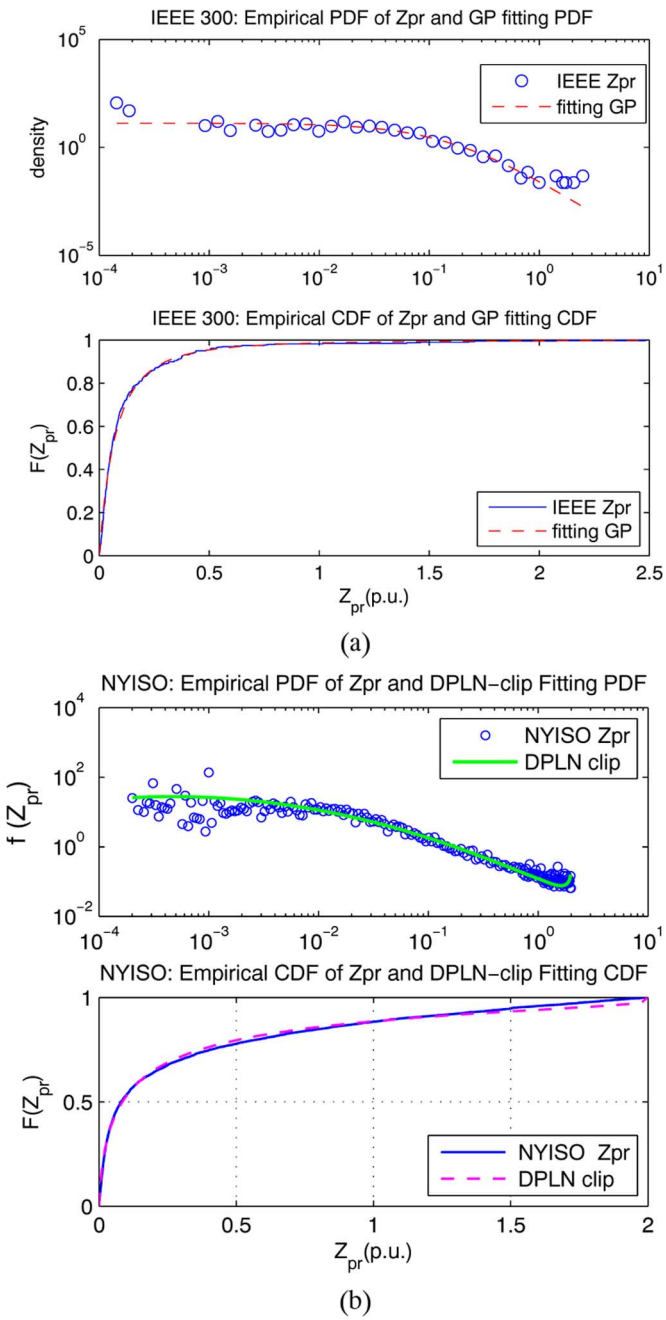


Fig. 9. Comparison between empirical pdf/CDF distributions and fitting distributions. (a) IEEE 300. (b) NYISO 2935.

of (α, β), as shown in Fig. 10, to select clusters of nodes and therefore groups of links to be rewired. This mechanism is introduced in order to produce a correlation among the rewired links.

After running the above Markov transition for N times (i.e., one for each node in the network), we get clusters of nodes with "1"s alternating with clusters of nodes with "0"s, where "1" means to rewire and "0" means not. Then by a specific rewiring probability $q_{\rm rw}$, some links are selected to rewire from all the links originating from each 1-cluster of nodes; and the corresponding local links get rewired to outside 1-clusters. The average cluster size for rewiring nodes is $K_{\rm clst} = \frac{1}{\beta}$; and the steady-state probabilities are $p_0 = \frac{\beta}{\alpha+\beta}$, $p_1 = \frac{\alpha}{\alpha+\beta}$. Experiments are performed on the available real-world power grid

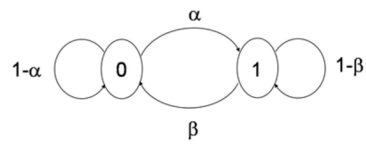


Fig. 10. Markov chain for selecting cluster of rewiring nodes (0: not rewire; 1: rewire).

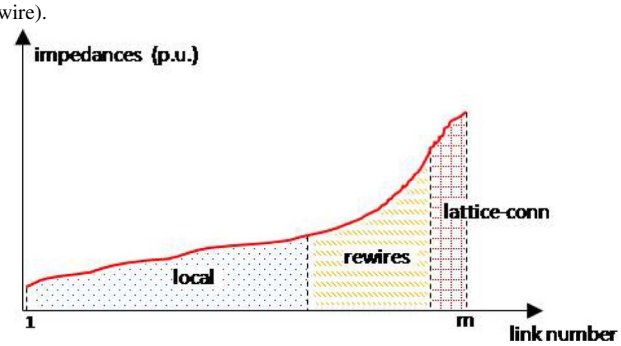


Fig. 11. Grouping line impedances.

data to estimate the parameters. The average rewiring cluster size $K_{\rm clst}$, the rewiring probability of links $q_{\rm rw}$, and the ratio of nodes having rewire links p_1 can be directly obtained from statistical estimates. Then the transition probability can be computed as $\beta = \frac{1}{K_{\rm clst}}$ and $\alpha = \frac{\beta p_1}{1-p_1}$.

B. Lattice Connections

In this step lattice connections are selected at random from neighboring subnetworks to form a whole large-scale power grid network, as shown in Fig. 10. The number of lattice connections between neighboring subnetworks is chosen to be an integer around $\langle k \rangle$.

C. Generation and Assignment of Line Impedances

In this part a number m of line impedances are generated from a specified heavy-tailed distribution, and then sorted by magnitude and group into local links, rewire links, and lattice connection links according to corresponding portions, as shown in Fig. 11. Finally, line impedances in each group are assigned at random to the corresponding group of links in the topology.

VI. Conclusion

In this work we have studied both the topological and electrical characteristics of electric power grids: the nodal degree distribution $\underline{k} = \mathrm{diag}(L)$, the line impedance distribution $Z_{\rm pr}$, the graph spectral density $\rho(\lambda)$, and the scaling law of the algebraic connectivity $\lambda_2(L)$, where L represents the Laplacian matrix of the network. It is found that the power grid is sparsely connected with a low average node degree which does not scale with the network size. We have provided evidence that the nodal degrees follow a mixture distribution which comes from the sum of a truncated geometric random variable and an irregular discrete random variable. We have also proposed a method to estimate the distribution parameters by analyzing the poles and zeros of the average probability generation function. We studied the graph spectral density and have shown that power grid network takes on a distinctive pattern for its graph spectral density. We studied the scaling law of the algebraic connectivity of power networks, and have shown that power networks are exceptionally well connected given their sparsity, featuring a

better scaling law than Watts–Strogatz *small-world* graphs. We also have shown that the distribution of line impedances Z_{pr} is heavy-tailed and can be captured quite accurately by a clipped DPLN distribution. Finally we have proposed a novel random topology power grid model, RT-*nested-Smallworld*, intending to capture the above observed characteristics.

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