Static State Estimation in Power Systems

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Energy Management System (EMS)

1. **Measurements**
   - Obtained from supervisory control and data acquisition (SCADA)
   - New data every 4 seconds

2. **Static state estimator**
   - Uses a model of the system that includes topology and parameter information
   - Estimates real-time values for the whole system
   - Signals to operators any lines close to overload or already overloaded
   - MISO runs this every 5 minutes
Static State Estimation

- Process to assign values to unknown system state variables based on imperfect measurements obtained from the system.

- State variables:
  1. Voltage magnitudes
  2. Relative phase angles at buses

- Measurements:
  1. Voltage magnitudes
  2. Real and reactive transmission line flows
  3. Current through transmission lines

- Objective: produce the “best estimate” of the system voltage and phase angles, recognizing that there are errors in the measured quantities and that there may be redundant measurements.

- Basic underlying assumption: power system is in quasi-steady-state condition.
Common criteria for “best” estimate

1. The maximum likelihood criterion: maximize probability that estimate of the state variable is its true value
2. The weighted least-squares criterion: minimize sum of squares of the weighted deviations of estimated measurements from actual measurements
3. The minimum variance criterion: minimize expected value of sum of squares of the deviations of estimated components of the state vector from the corresponding true values

Assume meter error distributions are normally distributed and unbiased

Then the three criteria above lead to identical estimators
Maximum Likelihood Least-Squares Estimation

- Let $x$ denote vector of state variables, i.e., voltage magnitudes and relative phase angles
- Let $\eta_i$ represent uncertainty in each measured value $z_{i}^{meas}$. Then,

$$z_{i}^{meas} = z_{i}^{true} + \eta_i = h_i(x) + \eta_i$$

- Model $\eta_i \sim \mathcal{N}(0, \sigma_i)$
- Assumptions:
  1. Zero-mean: $\mathbb{E}[\eta_i] = 0$, for all $i$
  2. Independent measurements:

$$\mathbb{E}[\eta_i \eta_j] = \begin{cases} \sigma_i^2 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$
Maximum Likelihood Least-Squares Estimation

- The p.d.f. of $z_i^{meas}$ is

$$f(z_i^{meas}) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left\{ -\frac{[z_i^{meas} - h_i(x)]^2}{2\sigma_i^2} \right\}$$

- Find estimate $\hat{x}$ that maximizes the probability that the observed measurement $z_i^{meas}$ would occur

  $\implies$ Find $\hat{x}$ that maximizes p.d.f. $f(z_i^{meas})$

  $\implies$ Find $\hat{x}$ that minimizes $\frac{[z_i^{meas} - h_i(x)]^2}{2\sigma_i^2}$

- Suppose there are $m$ measurements

- Leads to the following objective:

$$\min J(x) = \sum_{i=1}^{m} \frac{[z_i^{meas} - h_i(x)]^2}{2\sigma_i^2}$$
Matrix Form

- Let

\[
\begin{bmatrix}
z_1^{\text{meas}} \\
\vdots \\
z_m^{\text{meas}}
\end{bmatrix}, \quad h(x) = \begin{bmatrix}
h_1(x) \\
\vdots \\
h_m(x)
\end{bmatrix}, \quad R = \begin{bmatrix}
\sigma_1^2 \\
\vdots \\
\sigma_m^2
\end{bmatrix}
\]

- Weighted least-squares (WLS) estimation problem with \( R^{-1} \) as weighting matrix

\[
\min J(x) = \frac{1}{2} [z - h(x)]^T R^{-1} [z - h(x)] = J(\hat{x})
\]

- Necessary conditions of optimality are

\[
\nabla_x J|_{x=\hat{x}} = - [z - h(\hat{x})]^T R^{-1} \left[ \nabla_x h \right]_{x=\hat{x}} = 0
\]

- So \( \hat{x} \) is the root of the set of nonlinear equations

\[
H^T(x)R^{-1} [z - h(x)] = 0
\]
Weighted Least-Squares Estimation

\[ H^T(x)R^{-1} [z - h(x)] = 0 \]

- Can solve iteratively. Or...
- If each \( h_i \) is a linear function* of \( x \), i.e., \( h(x) = Hx \), then
  \[ H^T R^{-1} z - H^T R^{-1} Hx = 0 \]

- Closed-form solution exists:
  \[ \hat{x} = [H^T R^{-1} H]^{-1} H^T R^{-1} z \]

* DC power flow model assumptions
  
  - For each of the transmission lines, \( x_i >> r_i \).
  - All bus voltages are approximately 1 p.u.
  - All bus angles are nearly 0. Therefore, \( \cos(\theta_i - \theta_j) \approx 1 \) and \( \sin(\theta_i - \theta_j) \approx \theta_i - \theta_j \).
Example—Two-Bus System

4 measurements

\[
z = \begin{bmatrix} v_1 \\ v_2 \\ p_{12} \\ q_{12} \end{bmatrix} = \begin{bmatrix} 1.02 \\ 1.0 \\ 2.0 \\ 0.2 \end{bmatrix}, \quad R = \begin{bmatrix} 0.05^2 & 0 & 0 & 0 \\ 0 & 0.05^2 & 0 & 0 \\ 0 & 0 & 0.1^2 & 0 \\ 0 & 0 & 0 & 0.1^2 \end{bmatrix}
\]

3 states

\[
x = \begin{bmatrix} V_1 \\ V_2 \\ \theta_2 \end{bmatrix}
\]
Example—Two-Bus System

\[ y_{12} = -j10 \]

- **Measured values**

\[
z = h(x) = \begin{bmatrix} v_1 \\ v_2 \\ p_{12} \\ q_{12} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -10V_1V_2 \sin \theta_2 \\ 10V_1^2 - 10V_1V_2 \cos \theta_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}
\]

- **Jacobian**

\[
H(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -10V_2 \sin \theta_2 & -10V_1 \sin \theta_2 & -10V_1V_2 \cos \theta_2 \\ 20V_1 - 10V_2 \cos \theta_2 & -10V_1 \cos \theta_2 & 10V_1V_2 \sin \theta_2 \end{bmatrix}
\]

- **Iterative solution.** At the \( k^{th} \) iteration, solve for \( \Delta x_k \) in

\[
[H(x_k)]^T R^{-1} H(x_k) \Delta x_k = [H(x_k)]^T R^{-1} [z - h(x_k)]
\]