Opening the GPS Blackbox!

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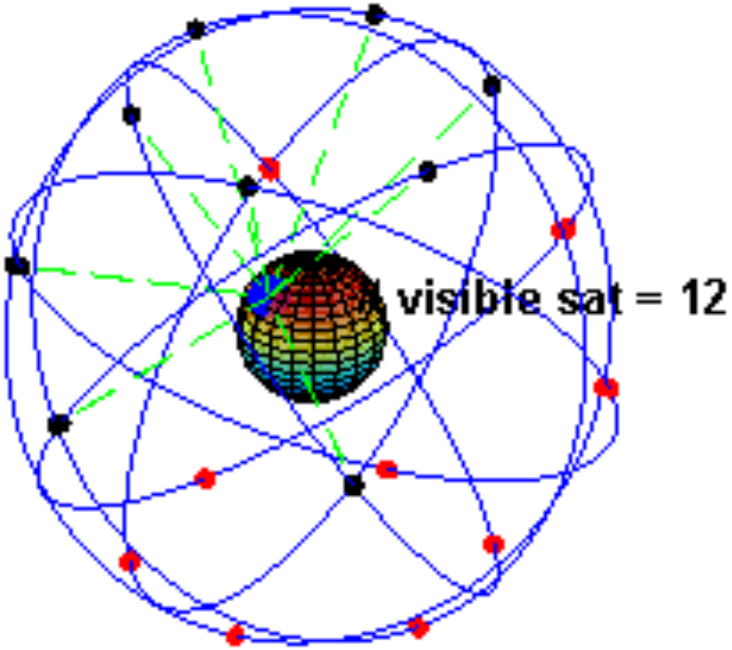
Global Positioning System (GPS)



Matlab simulation of the GPS constellation at 2880 times faster than real time.
uploaded onto Wikipedia in 2007 by El Pak

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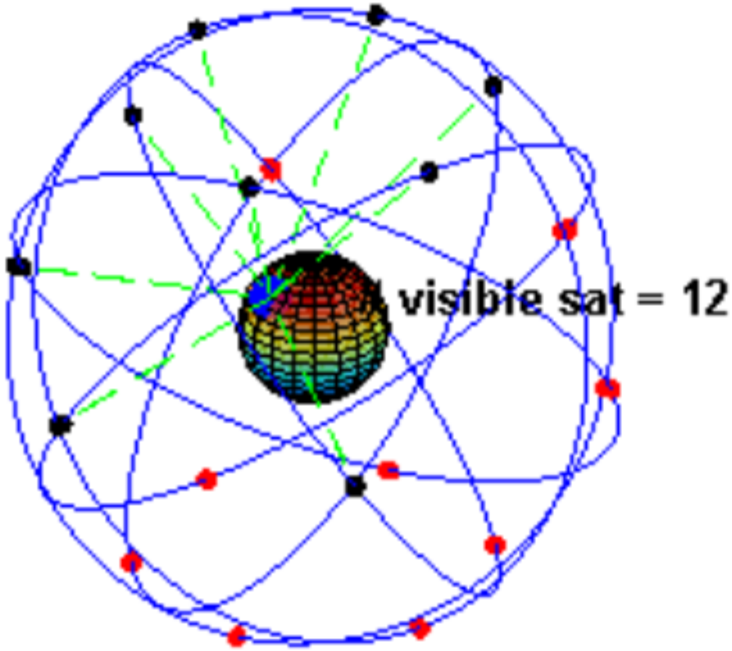
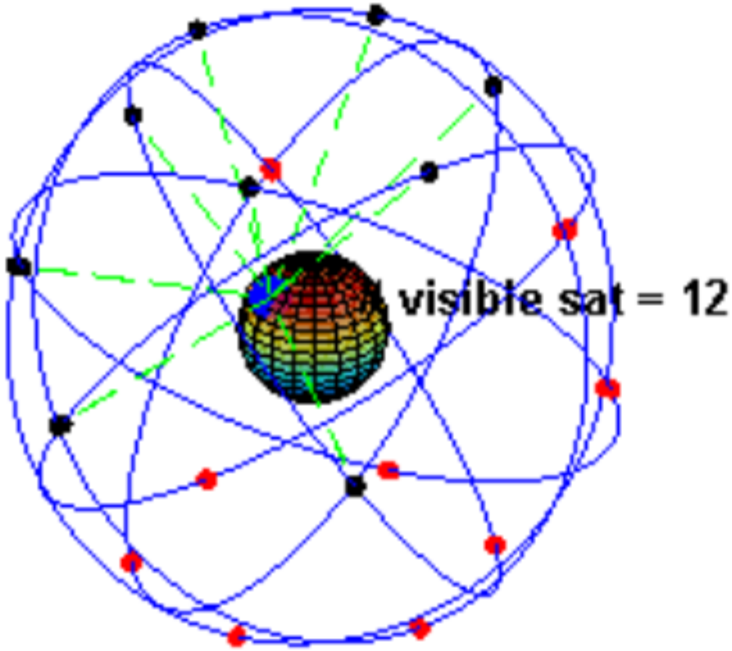


- Satellite navigation system

Global Positioning System (GPS)



- Satellite navigation system
- Positioning coordinates on Earth

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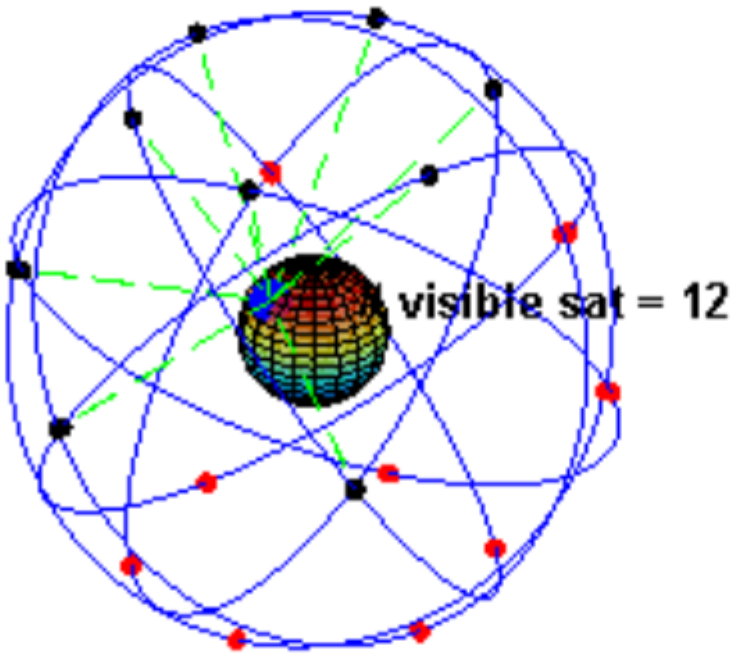
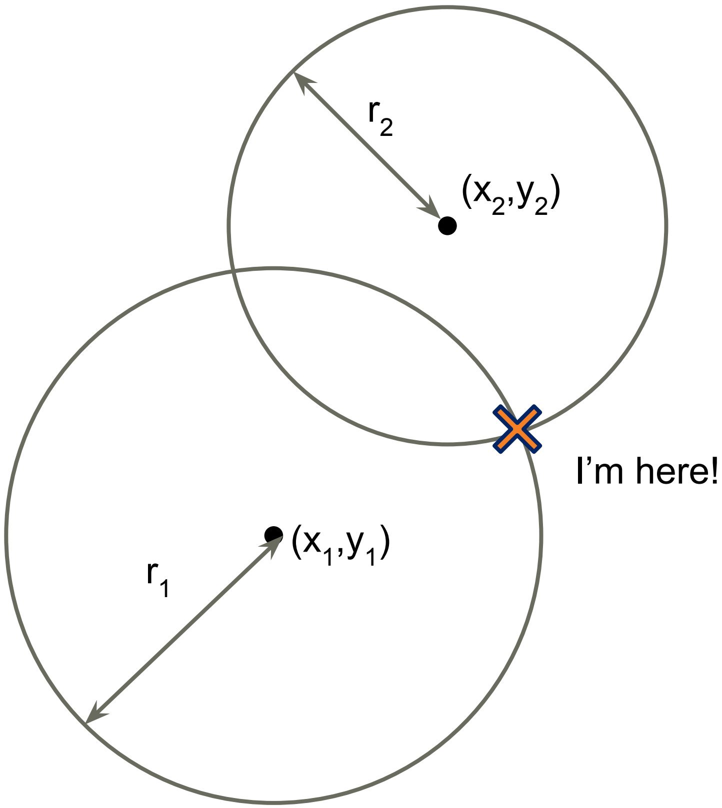


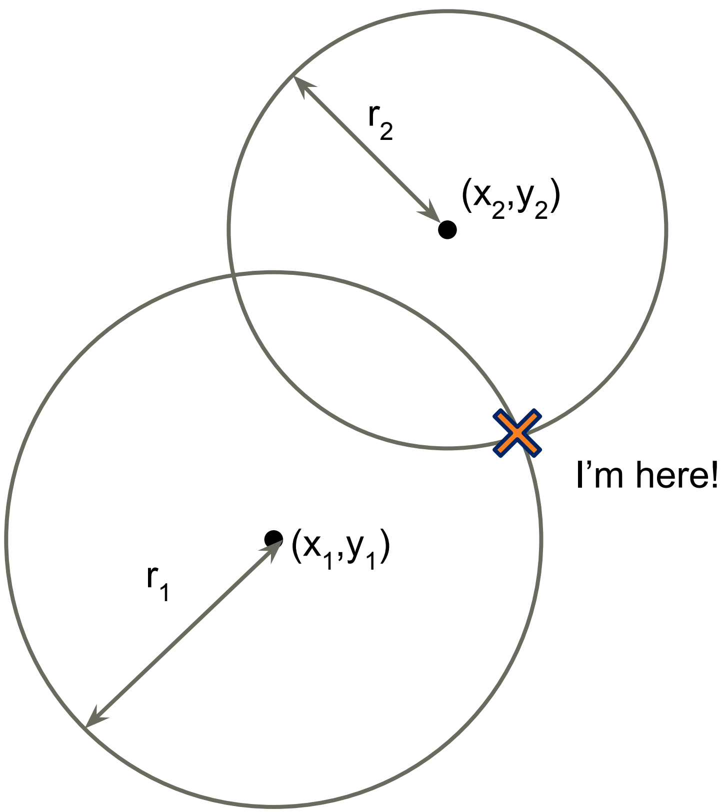
- Satellite navigation system
- Positioning coordinates on Earth
- How is that done?

Trilateration



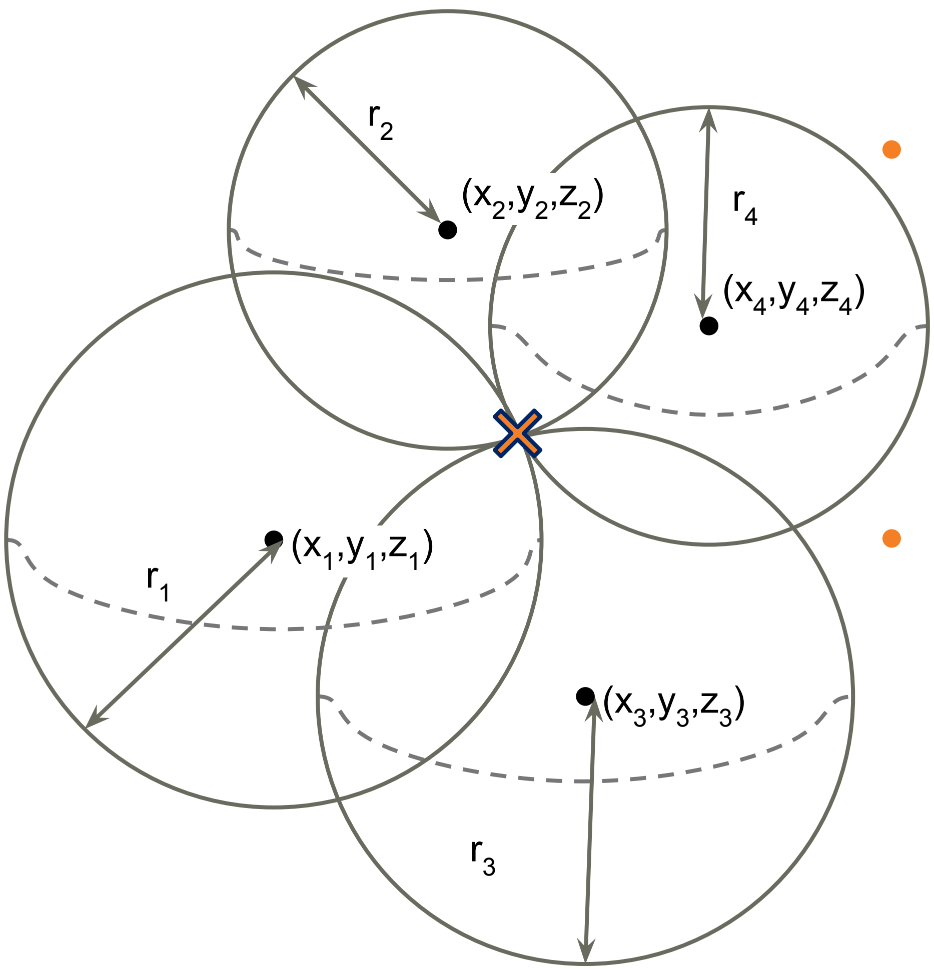
- 2D example
- $$(x_1 - x)^2 + (y_1 - y)^2 = r_1^2$$
  $$(x_2 - x)^2 + (y_2 - y)^2 = r_2^2$$
- Solve for (x, y)

Trilateration



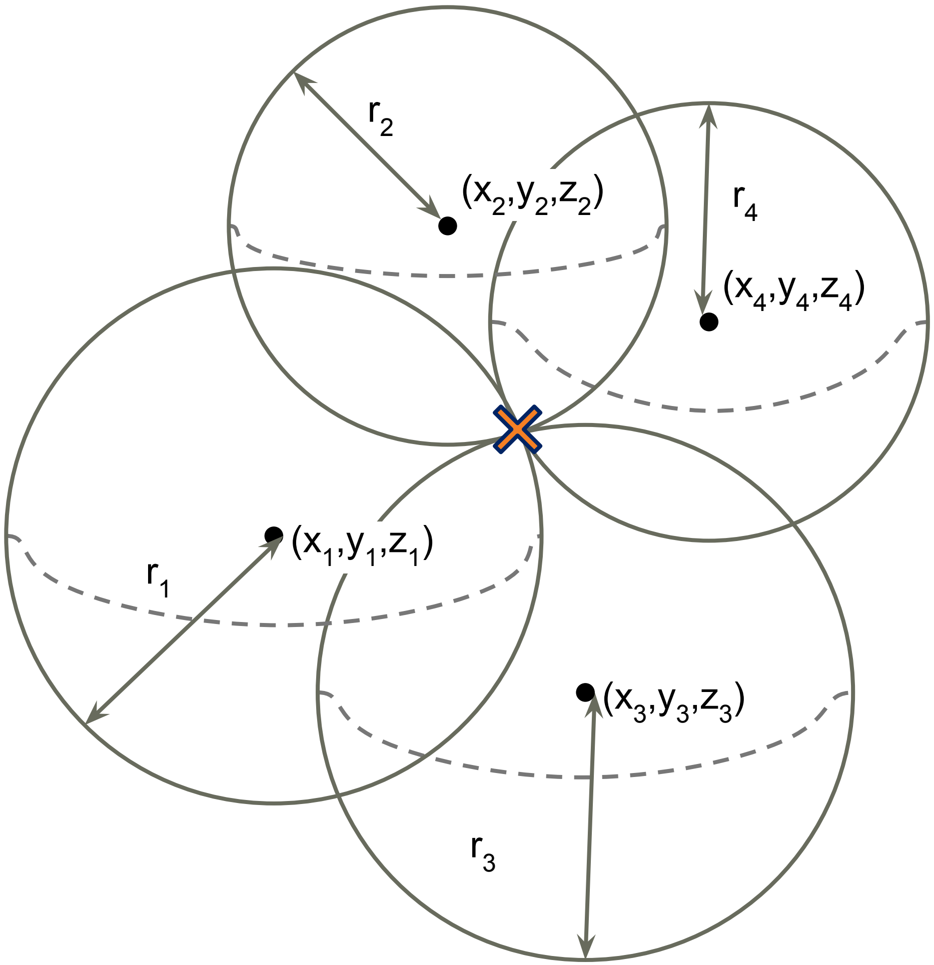
- 2D example
- $$(x_1 - x)^2 + (y_1 - y)^2 = r_1^2$$
  $$(x_2 - x)^2 + (y_2 - y)^2 = r_2^2$$
- How to generalize to 3D?

Trilateration



- 3D example
- Overdetermined system → Least-squares
- Iterate using Newton-Raphson

Trilateration



Need to know:

- Satellite Positions
- Satellite Ranges

Satellite Signals

- Satellite position : broadcast message
- Satellite range : ranging signal

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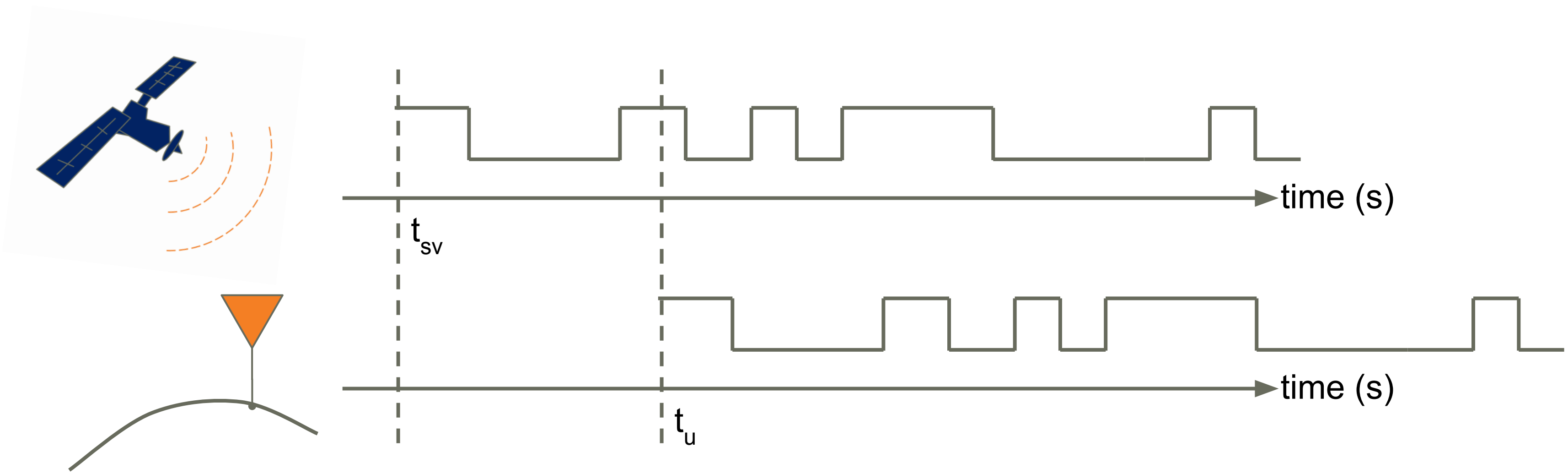
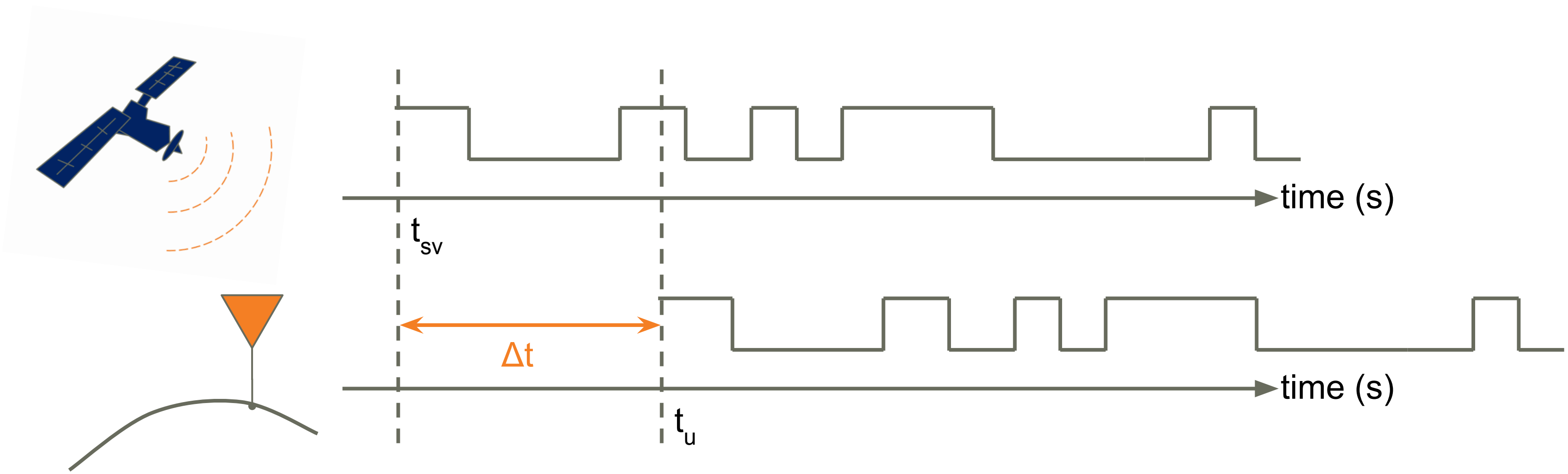


sv : **s**pace **v**ehicle; satellite; in the sky
u : **u**ser; receiver; on the ground

Satellite Signals

- Satellite position : broadcast message
- Satellite range : ranging signal



Δt = travel time bt. sv and u

$c\,\Delta t$ = range bt. sv and u

sv : **s**pace **v**ehicle; satellite; in the sky
u : **u**ser; receiver; on the ground

Timing Complexity

- most user receiver clocks are inaccurate
- 1ms time error ≈ 300,000m range error

Timing Complexity

- most user receiver clocks are inaccurate
- model timing errors:

$$\widetilde{t_{sv}} = t_{sv} + \delta t_{sv}$$

$$\widetilde{t_u} = t_u + \delta t_u$$

measured user time — actual user time — error in user time

Timing Complexity

$$c(\widetilde{\Delta t_u}) = c((t_u + \delta t_u) - (t_{sv} + \delta t_{sv}))$$

measured time difference

measured user time

measured satellite time

$$c(\widetilde{\Delta t_u}) = c(t_u - t_{sv}) + c\delta t_u - c\delta t_{sv}$$

$$\rho = c(t_u - t_{sv}) + c\delta t_u$$

pseudorange

true range

error in user time

Equation Formulation

$$\underbrace{\rho}_{\text{pseudorange}} = \underbrace{c(t_u - t_{sv})}_{\text{true range}} + \underbrace{c\delta t_u}_{\text{error in user time}}$$

pseudorange

true range

error in
user time

Equation Formulation

$$\rho = c(t_u - t_{sv}) + c\delta t_u$$

pseudorange — true range — error in user time

$$\rho^1 = ((x^1 - x_u)^2 + (y^1 - y_u)^2 + (z^1 - z_u)^2)^{1/2} + c\delta t_u$$

$$\rho^2 = ((x^2 - x_u)^2 + (y^2 - y_u)^2 + (z^2 - z_u)^2)^{1/2} + c\delta t_u$$

$$\rho^3 = ((x^3 - x_u)^2 + (y^3 - y_u)^2 + (z^3 - z_u)^2)^{1/2} + c\delta t_u$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\rho^n = ((x^n - x_u)^2 + (y^n - y_u)^2 + (z^n - z_u)^2)^{1/2} + c\delta t_u$$

Iterative Expression

$$
\begin{bmatrix} \rho^1 - \widehat{\rho}^1 \\ \rho^2 - \widehat{\rho}^2 \\ \rho^3 - \widehat{\rho}^3 \\ \cdots \\ \rho^n - \widehat{\rho}^n \end{bmatrix}
=
\begin{bmatrix}
\frac{-(x^1 - \widehat{x})}{\rho^1} & \frac{-(y^1 - \widehat{y})}{\rho^1} & \frac{-(z^1 - \widehat{z})}{\rho^1} & c \\
\frac{-(x^2 - \widehat{x})}{\rho^2} & \frac{-(y^2 - \widehat{y})}{\rho^2} & \frac{-(z^2 - \widehat{z})}{\rho^2} & c \\
\frac{-(x^3 - \widehat{x})}{\rho^3} & \frac{-(y^3 - \widehat{y})}{\rho^3} & \frac{-(z^3 - \widehat{z})}{\rho^3} & c \\
\cdots & \cdots & \cdots & \cdots \\
\frac{-(x^n - \widehat{x})}{\rho^n} & \frac{-(y^n - \widehat{y})}{\rho^n} & \frac{-(z^n - \widehat{z})}{\rho^n} & c
\end{bmatrix}
\begin{bmatrix} (x - \widehat{x}) \\ (y - \widehat{y}) \\ (z - \widehat{z}) \\ (\delta t_u - \widehat{\delta t_u}) \end{bmatrix}
$$

residuals

geometry matrix

updates

Iterative Expression

$$\begin{bmatrix} \rho^1 - \widehat{\rho}^1 \\ \rho^2 - \widehat{\rho}^2 \\ \rho^3 - \widehat{\rho}^3 \\ \cdots \\ \rho^n - \widehat{\rho}^n \end{bmatrix} = \begin{bmatrix} \frac{-(x^1-\widehat{x})}{\rho^1} & \frac{-(y^1-\widehat{y})}{\rho^1} & \frac{-(z^1-\widehat{z})}{\rho^1} & c \\ \frac{-(x^2-\widehat{x})}{\rho^2} & \frac{-(y^2-\widehat{y})}{\rho^2} & \frac{-(z^2-\widehat{z})}{\rho^2} & c \\ \frac{-(x^3-\widehat{x})}{\rho^3} & \frac{-(y^3-\widehat{y})}{\rho^3} & \frac{-(z^3-\widehat{z})}{\rho^3} & c \\ \cdots & \cdots & \cdots & \cdots \\ \frac{-(x^n-\widehat{x})}{\rho^n} & \frac{-(y^n-\widehat{y})}{\rho^n} & \frac{-(z^n-\widehat{z})}{\rho^n} & c \end{bmatrix} \begin{bmatrix} (x - \widehat{x}) \\ (y - \widehat{y}) \\ (z - \widehat{z}) \\ (\delta t_u - \widehat{\delta t_u}) \end{bmatrix}$$

b   A   Δx

$$\Delta x = (A^\top A)^{-1} A^\top b$$

$$x = \widehat{x} + \Delta x$$

GPS Applications

$$\begin{bmatrix} \underbrace{x \quad y \quad z}_{\text{user position}} & \underbrace{\delta t_u}_{\text{user time bias}} \end{bmatrix}$$

GPS Applications

$$\begin{bmatrix} x & y & z & \delta t_u \end{bmatrix}$$

Navigation (x, y, z)

Time Transfer (δt_u)

- Communications
- Electrical power grids
- Financial networks
- ...

GPS Applications

$$\begin{bmatrix} x & y & z & \delta t_u \end{bmatrix}$$

Navigation ($x\ y\ z$)

Time Transfer (δt_u)

- Communications
- Electrical power grids
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- ...

precise and accurate time synchronization

Timing Requirements for Power Grid

- IEEE C37.118 Standard
  - timing agreement between PMU measurements < 31.8μs or < 26.5μs for 50Hz or 60Hz systems
- Precise Time Protocol (PTP) tool in IEEE 1588 gives ~μs precision
- GPS time synchronization ~ns precision

http://standards.ieee.org/findstds/standard/C37.118.1-2011.html

Timing Measurements

- sampling synchronization
- Fault Locators (FL) with time tags
- …

System Function	Measurement	Optimum Accuracy	Time Sources
TW Fault Locator	300 m (line span)	1 μs	GPS
Relaying (line protection)	1000 m	3 μs	GPS
Phasor Measurement	+/- 0.1 degree	4.6 μs (60 Hz)	GPS
Networked Controls	+/- 0.1 degree	4.6 μs	GPS
Stability Controls (RAS,etc)	+/- 1 degree	46 μs	GPS
Event recording (DFR, etc)	Record compare	1 ms	GPS
Generation Control (AGC)	Freq, time error	10 ms	GPS, Net
Scheduling, reservation	Time of day	0.5 sec	GPS, Net

http://www.unoosa.org/pdf/icg/2010/ICG5/timing-session/01.pdf

Research Work

- Sensor fusion
- Measurement level integration
- Signal level integration
  - Multi-receiver position-information-aided vector tracking for robust timing of PMUs
  - Multi-receiver vector tracking with software defined radio in Python
- Vector correlator
- Swarm navigation test bench

Thank You!

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