Computation of Margins to Power System Loadability Limits Using Phasor Measurement Unit Data (or the SDG conjecture)

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Limits to Power System Operation (sources of congestion)

- Thermal – short term and long term – typically measured in Amps or power (MW or MVA) – this one is fairly easy to find from measurements.
- Voltage – plus or minus 5% of nominal – this one is fairly easy to find from measurements.
- Stability – voltage collapse, SS stability, transient stability, bifurcations – margins to each critical point – this one is hard to find.
- Other
  - Control limits - Ramp constraints, under/over excitation, taps
  - Short circuit current capability

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Transmission Lines
230 kV steel tower double circuit

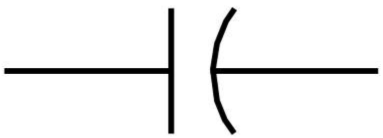
What is loadability?

- You might think:
  - For one voltage level and one current limit, there is a power limit proportional to the product.
  - If you double the voltage, and use the same conductors (same current limit), then the loadability should double. This is sort of correct for the thermal limit.

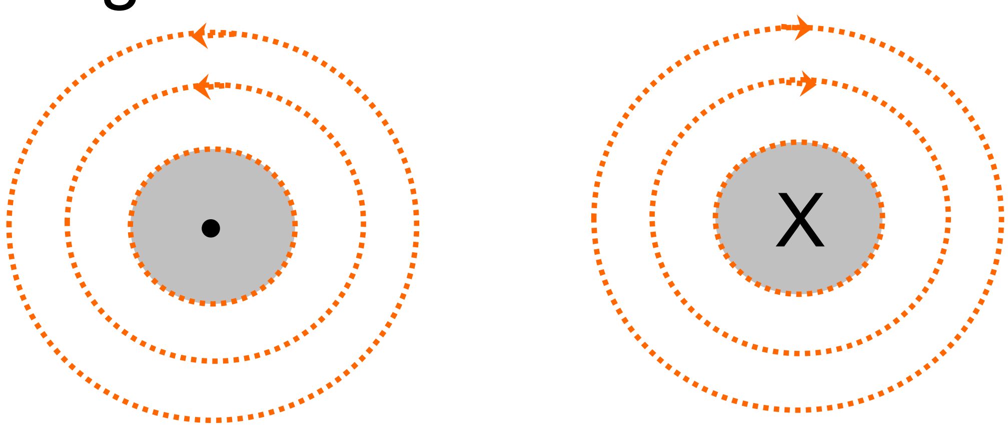
- Actually:
  - When you consider all limiting phenomena, the loadability goes up more as the square of the voltage.

Transmission Line Parameters

- Electric fields "due to voltage“ — Model as C

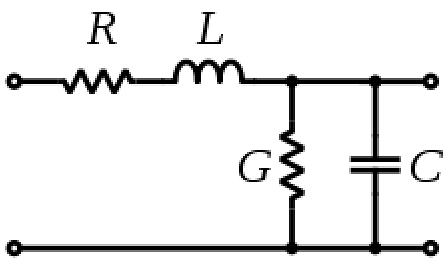


- Magnetic fields "due to current “ — Model as L



Transmission Line Segment Model

L (Henries “per mile”) and C (Farads “per mile”)



“A 200 mile line has 200 of these segments”

Characteristic impedance of a lossless line is $\sqrt{L/C}$ Ohms

Surge Impedance Load (SIL)

- 1.0 SIL is the power delivered by a “lossless” line to a load resistance equal to the surge (characteristic) impedance = $\sqrt{L / C}$ Ohms (typically 300 to 400 Ohms)

- Flat voltage profile along entire line

- Voltage and current are in phase along entire line

- VARS into line from shunt charging are exactly equal to the total line VAR series losses

Ballpark values of 1.0 SIL

$$SIL = V^2/R_c$$

69 KV	$\longrightarrow$	12 MW
138 KV	$\longrightarrow$	50 MW
230 KV	$\longrightarrow$	140 MW
345 KV	$\longrightarrow$	400 MW
500 KV	$\longrightarrow$	1000 MW
765 KV	$\longrightarrow$	2000 MW

St. Clair and AEP curves

[1] H. P. St. Clair, "Practical Concepts in Capability and Performance of Transmission Lines," AIEE Transactions (Power Apparatus and Systems). Paper 53-338 presented at the AIEE Pacific General Meeting, Vancouver, B. C., Canada, September 1-4, 1953.

[2] R.D. Dunlop, R. Gutman, P.Marchenko, "Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No.2 March/April 1979.

[3] Richard Gutman, “Application of Line Loadability Concepts to Operating Studies,” IEEE Transactions on Power Systems, Vol. 3, No. 4, November 1988.

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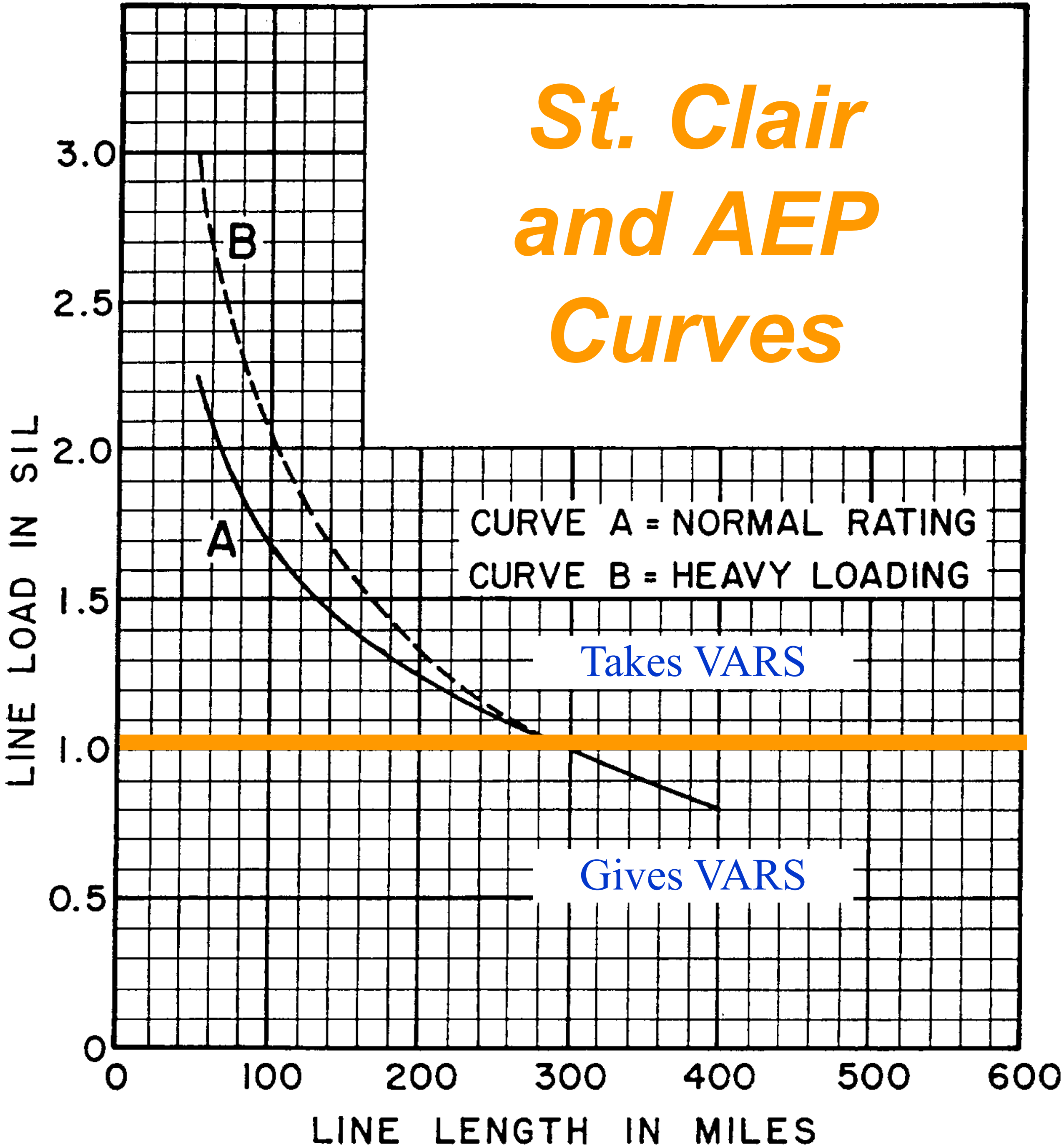
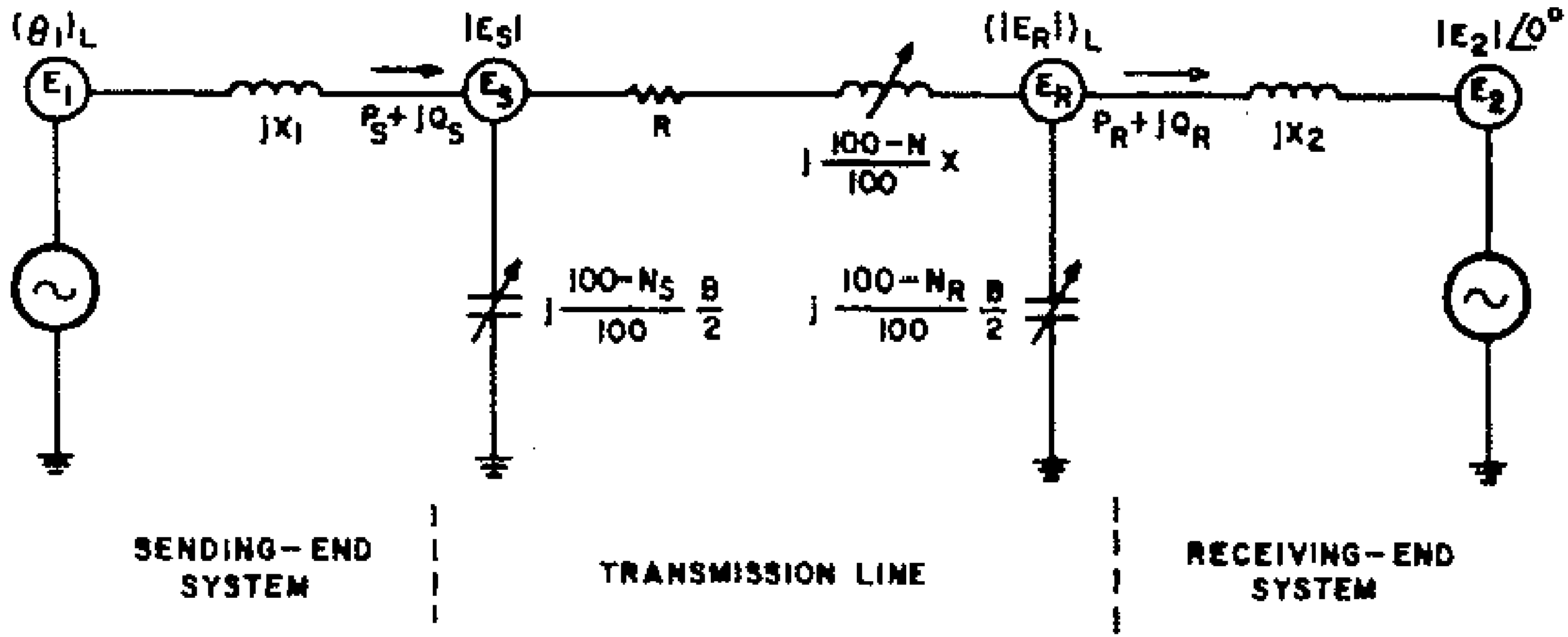


R.D. Dunlop, R. Gutman, P.Marchenko, "Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No.2 March/April 1979. Gutman was/is with AEP.



Power systems 101

Real power transmitted across a lossless line:

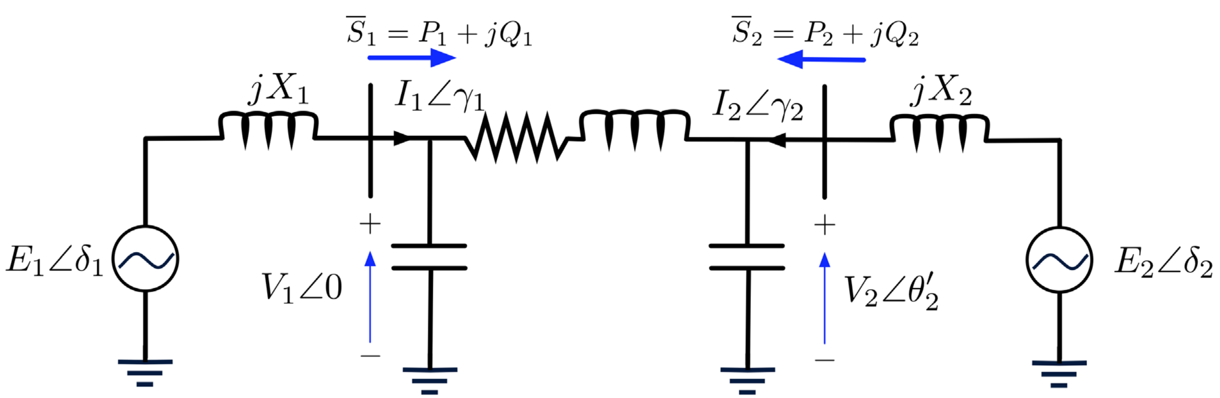
$$P_{12} = \frac{V_1 V_2}{X_{12}} \sin(\delta_1 - \delta_2)$$

And typically V_1 and V_2 are near nominal.

Series compensation is used to reduce X_{12}

SDG Conjecture

If you compute a Thevenin Equivalent as seen by both ends of a transmission line, the angle across the system will indicate a level of loading in the system – and this angle should approach 90 degrees at the critical line/equivalent combination. At 45 degrees there would be a 30% margin.

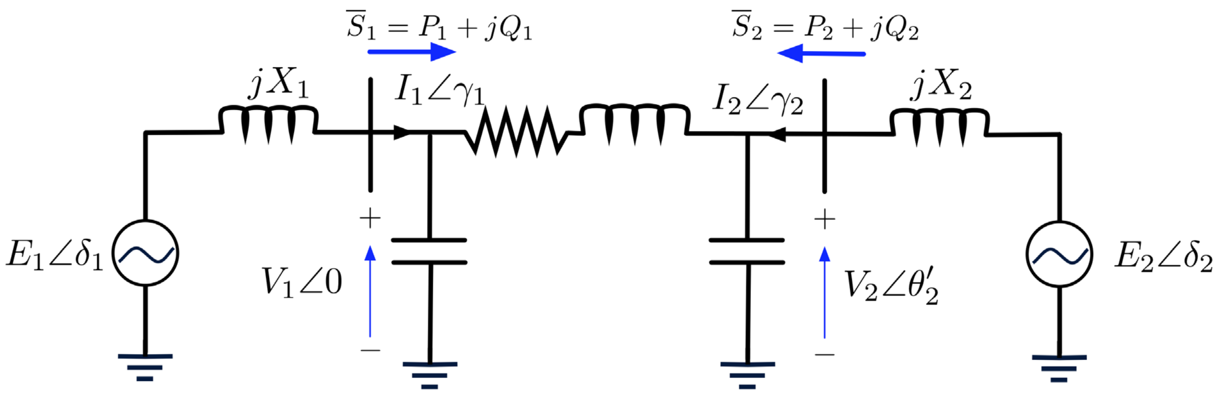


Others

- St. Clair and AEP curves

- T. He, S. Kolluri, S. Mandal, F. Galvan, P. Rastgoufard, "Identification of Weak Locations using Voltage Stability Margin Index", APPLIED MATHEMATICS FOR RESTRUCTURED ELECTRIC POWER SYSTEMS – Optimization, Control, and Computational Intelligence, Edited by Joe H. Chow, Felix F. Wu, James A. Momoh, Springer, 2005, p. 25 -37. This was done for Entergy.

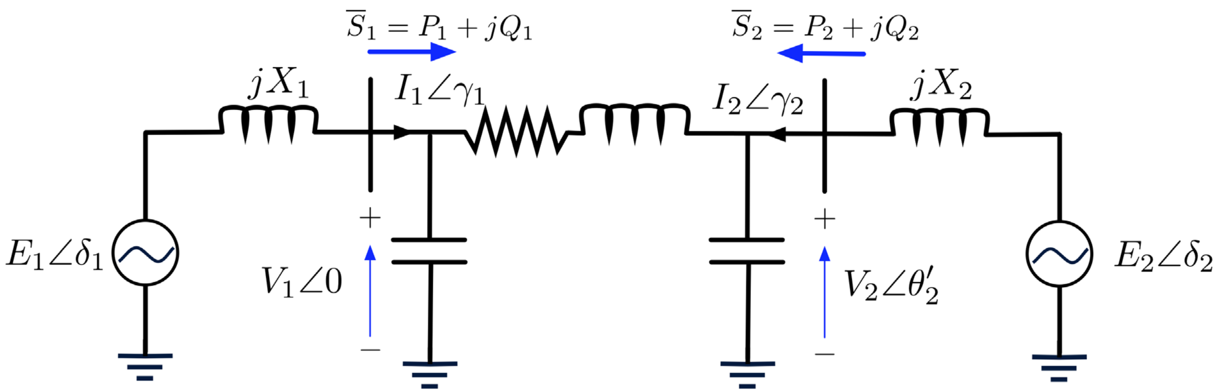
PMU data



- In the above line plus equivalents, PMU measurements at both ends will provide voltages V_1, V_2, (magnitude and angle) and currents I_1 and I_2 (magnitude and angle)

- From these measurements, we only need to compute the angle difference $\delta_1 - \delta_2$ (we really don't care about E or X)

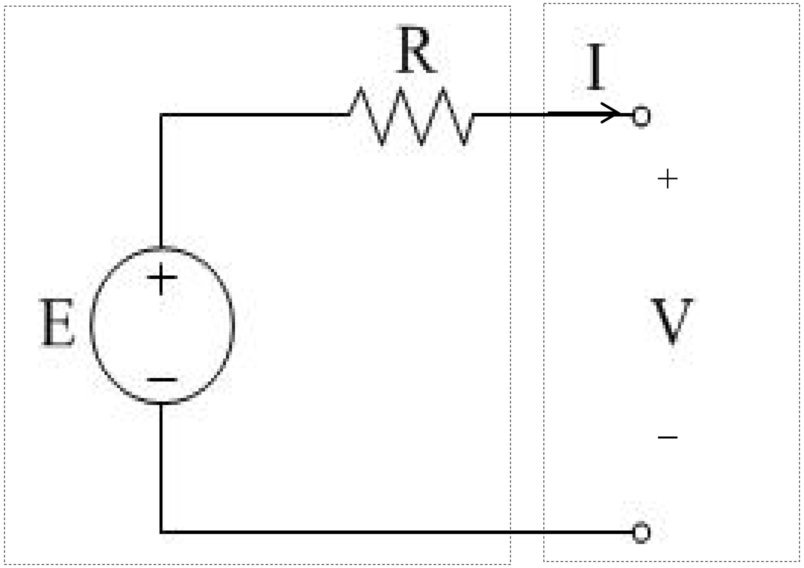
Simplify



- Ignore the shunt capacitance for now and notice that the two Thevenin reactances and voltages are in series – there is then only one current and one voltage to be measured.
- The Thevenin voltage is the series difference of the two and the Thevenin reactance is the series sum of the two.

Root computation (DC version)

Thévenin Equivalent Measurements



$E = IR + V$

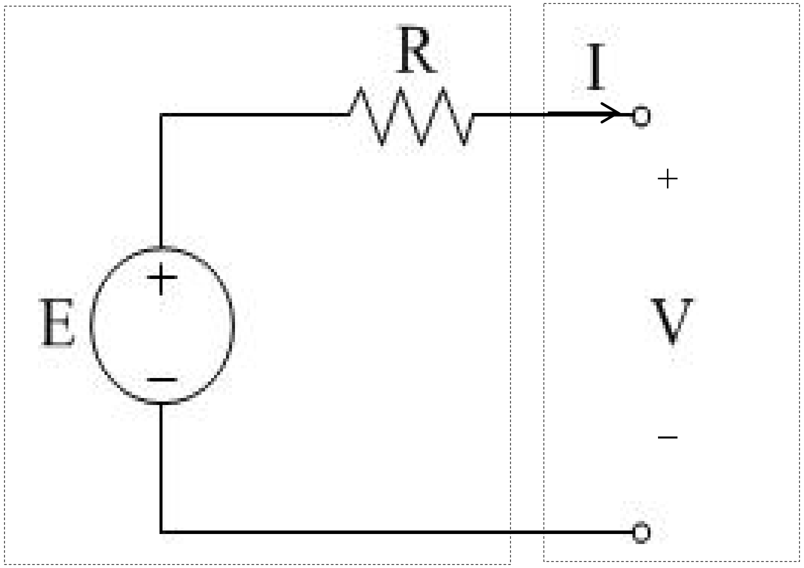
Measure V and I

Compute E and R

It doesn't get much easier than this?

Two consecutive measurements

Thévenin Equivalent Measurements



Assume that E and R do not change

$$E = I_1 R + V_1$$
$$E = I_2 R + V_2$$

Solve for E and R

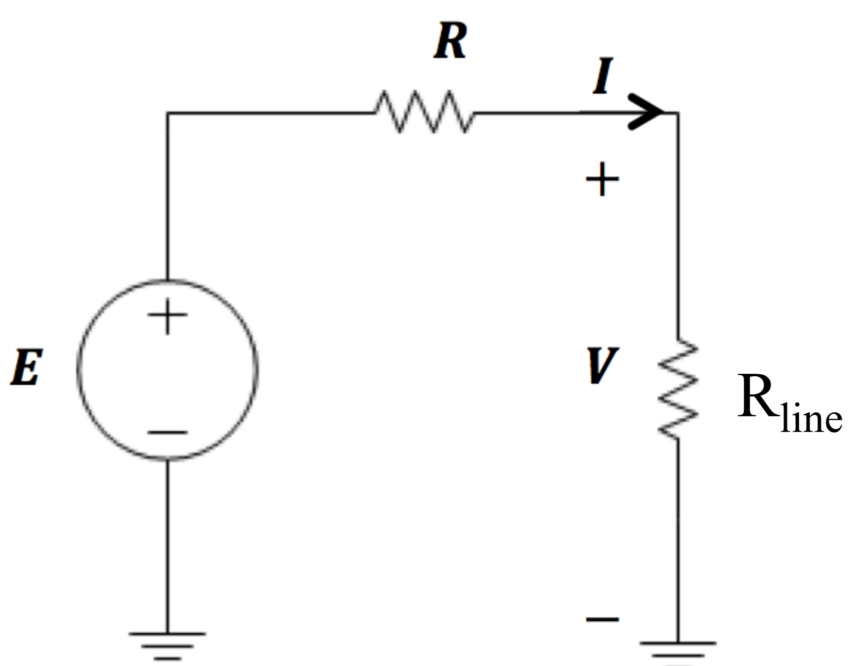
Two by two

$$\begin{bmatrix} 1 & -I_1 \\ 1 & -I_2 \end{bmatrix} \begin{bmatrix} E \\ R \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hopefully $\mathbf{I_1}$ and $\mathbf{I_2}$ are sufficiently different.

It would not take a significant modification of the measurements to create a solvability problem.

Suppose V and I are related

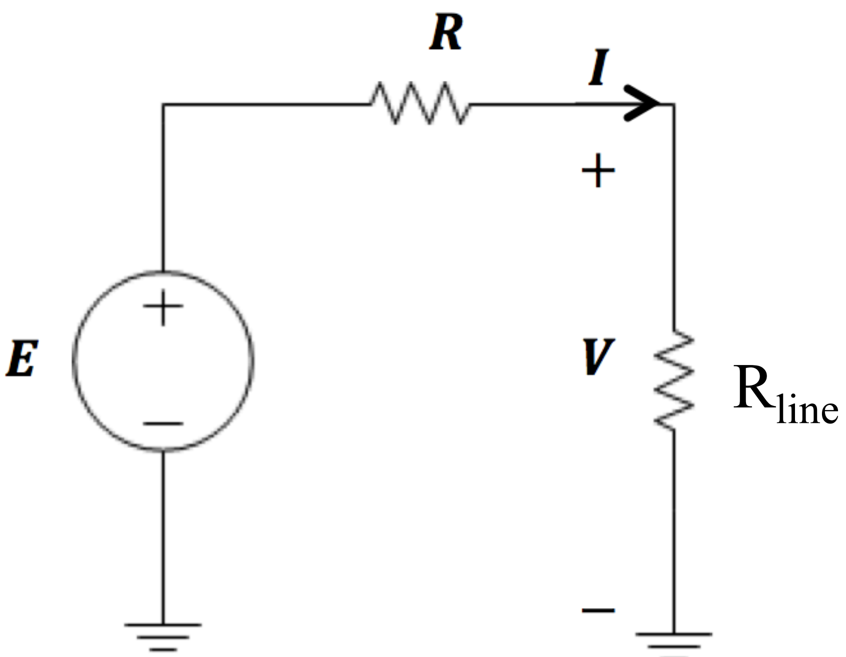


Assume R_{line} is known and does not change (remember, it is "the line")

If R_{line} does not change, then I cannot change, so no solution is possible.

But, what if I changes because E and/or R change while R_{line} does not change?

One algorithm



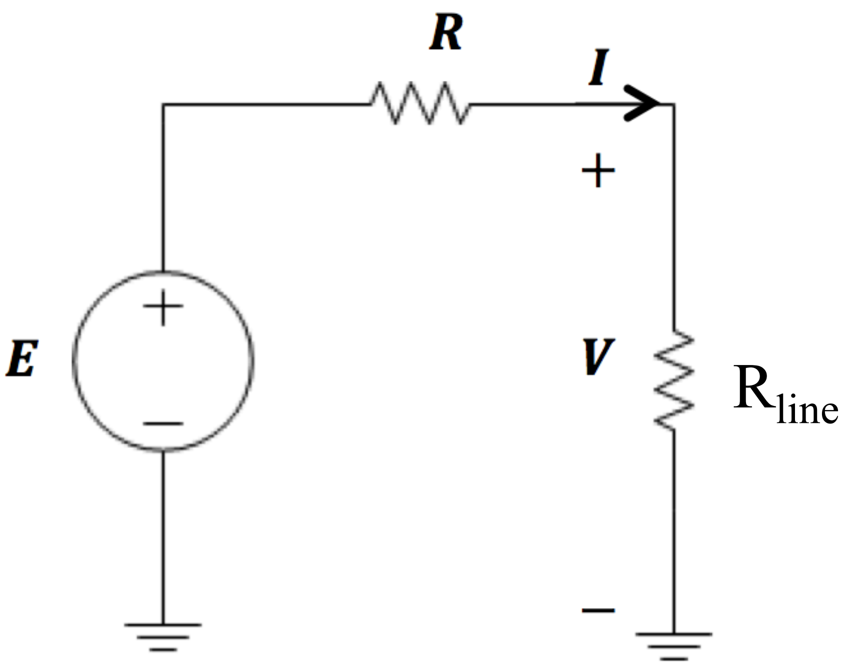
Don't worry about if R_{line} changes or not

Don't worry about if E changes or not (you don't know or care what it is anyway)

Don't worry about if R changes or not (you don't know or care what it is anyway)

Just use the two measurements to compute E and R if you can.

Another algorithm



Take more than two measurements in sequence (V_1, I_1, V_2, I_2, V_3, I_3) and do a some kind of best estimate of E and R.

The AC case

- The measurements of line voltage V and current I are complex numbers – fundamental frequency phasors.
- The Thevenin equivalent has a complex E and Z.
- Still a two-by-two problem, just with complex numbers.

Real data example

- The set of measured quantities include
  - Line-to-line voltages at both ends of the line
  - 3-phase complex power flowing into both ends of the line
- Measured quantities are sampled ten times per second
- Pseudo-measurements of line currents are obtained from the relation between complex power, voltage, and current
- Least Squares Errors (LSE) estimation is used to obtain per-second estimates of measurements and pseudo-measurements
- Since the system is at off-nominal frequency, phasor measurements rotate at a speed equal to the difference between the actual system frequency and the nominal frequency
  - To compensate for this effect, voltage estimates are redefined by defining the angle on one of the line ends to be zero and adjusting all other angles accordingly

Real data

19:08:01	761.27	-110.86	59.995
19:08:02	761.33	-111.03	59.996
19:08:02	761.27	-111.19	59.996
19:08:02	761.28	-111.34	59.996
19:08:02	761.16	-111.49	59.996
19:08:02	761.13	-111.64	59.996
19:08:02	NaN	NaN	NaN
19:08:02	NaN	NaN	NaN
19:08:02	761.09	-112.06	59.996
19:08:02	760.99	-112.19	59.996
19:08:02	760.92	-112.33	59.996
19:08:03	760.86	-112.48	59.996
19:08:03	760.9	-112.62	59.996
19:08:03	760.91	-112.77	59.996
19:08:03	761.03	-112.89	59.997
19:08:03	760.94	-113.02	59.997
19:08:03	760.89	-113.12	59.997
19:08:03	760.9	-113.22	59.997
19:08:03	760.98	-113.32	59.997
19:08:03	760.99	-113.43	59.997
19:08:03	761.09	-113.54	59.997
19:08:04	761.16	-113.66	59.997

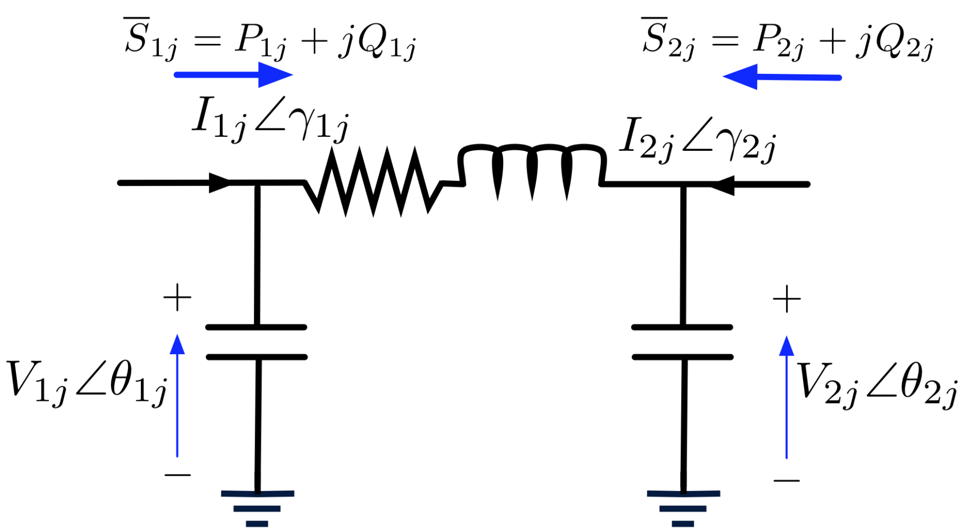
19:08:20	760.63	-122.96	60
19:08:20	760.63	-122.96	60
19:08:20	760.74	-122.96	60
19:08:20	760.78	-122.95	60
19:08:20	760.78	-122.95	60
19:08:20	760.9	-122.93	60
19:08:20	760.83	-122.93	60
19:08:20	760.92	-122.9	60
19:08:20	760.97	-122.89	60
19:08:20	760.97	-122.87	60
19:08:21	761.02	-122.86	60.001
19:08:21	760.93	-122.85	60
19:08:21	760.96	-122.82	60.001
19:08:21	761.03	-122.77	60.001
19:08:21	761.02	-122.71	60.002
19:08:21	761.03	-122.63	60.002
19:08:21	760.92	-122.53	60.002
19:08:21	760.83	-122.42	60.003
19:08:21	760.75	-122.31	60.003
19:08:22	760.73	-122.19	60.003
19:08:22	760.68	-122.08	60.003
19:08:22	760.69	-121.99	60.003

Per-Second Voltage Estimate

- Phasor voltages measured on ends 1 and 2:

$$\overline{V}_{1j} = V_{1j}\angle\theta_{1j}$$
$$\overline{V}_{2j} = V_{2j}\angle\theta_{2j}$$

where $j=1,2,\ldots,10$ indexes the samples taken every second



- Per-second voltage estimate:

$$\hat{V}_1 = \frac{\sum_{j=1}^{10} V_{1j}}{10}, \ \hat{\theta}_1 = 0,$$
$$\hat{V}_2 = \frac{\sum_{j=1}^{10} V_{2j}}{10}, \ \hat{\theta}_2' = \frac{\sum_{j=1}^{10} \theta_{2j}}{10} - \frac{\sum_{j=1}^{10} \theta_{1j}}{10}.$$

where voltage magnitudes are line-to-neutral

- Active and reactive power are per-phase

Thevenin Parameter Estimation

- Let E_1 and E_2 denote the Thevenin voltages on ends 1 and 2 of the line respectively, and let δ_1 and δ_2 be the Thevenin voltage source angles
- Let X_1 and X_2 be the corresponding Thevenin reactances (using R=0).
- Per-second estimates can be obtained as follows (using rated E):

$$\hat{\delta}_1[i] = \frac{1}{N}\sum_{j=1}^{N}\left(\gamma_1[i,j] + \arccos\left(\frac{V_{1\phi}[i,j]\cos(0-\gamma_1[i,j])}{E_1}\right)\right)$$

$$\hat{\delta}_2[i] = \frac{1}{N}\sum_{j=1}^{N}\left(\gamma_2[i,j] + \arccos\left(\frac{V_{2\phi}[i,j]\cos(\theta_2'[i,j]-\gamma_2[i,j])}{E_2}\right)\right)$$

$$\hat{X}_1[i] = \frac{1}{\sum_{j=1}^{N} I_1^2[i,j]}\sum_{i=1}^{N} I_1[i,j](\sqrt{E_1^2 - V_{1\phi}^2[i,j]\cos^2(0-\gamma_1[i,j])} - V_{1\phi}[i,j]\sin(0-\gamma_1[i,j]))$$

$$\hat{X}_2[i] = \frac{1}{\sum_{j=1}^{N} I_2^2[i,j]}\sum_{i=1}^{N} I_2[i,j]\sqrt{E_2^2 - V_{2\phi}^2[i,j]\cos^2(\theta_2'[i,j]-\gamma_2[i,j])} - V_{2\phi}[i,j]\sin(\theta_2'[i,j]-\gamma_2[i,j]))$$

Stability Reliability Measure

- Let δ_{12}^{max} be the maximum angle-across-system that ensures acceptable small-signal stability margin – i.e. 45 degrees
- A per-second stability reliability index (i indexes seconds) can be defined as:

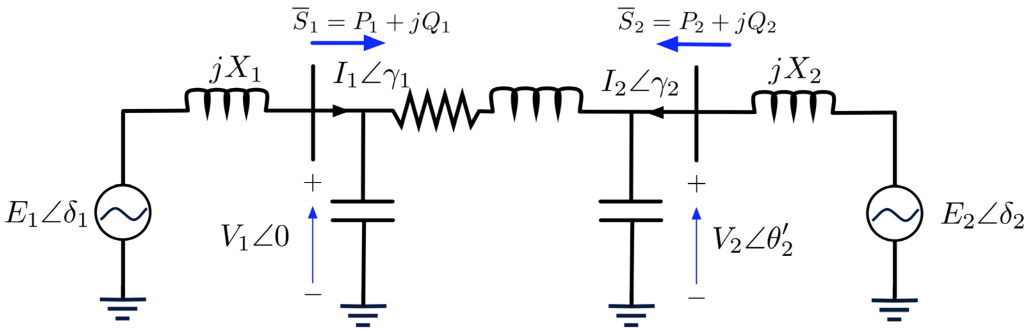
$$L^s[i] = 1 - \frac{\delta_{12}^{max} - |\hat{\delta}_{12}[i]|}{\delta_{12}^{max}}$$

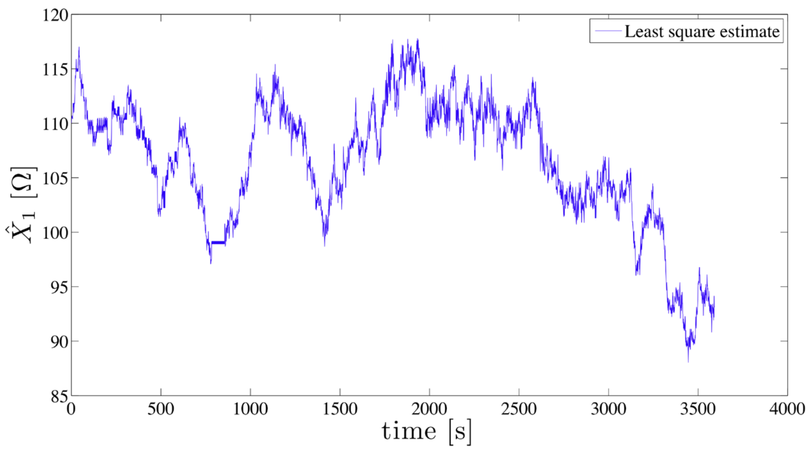
- These per-second indices are the basis for defining stability reliability measures
- For a one-hour period:
  - Normalized stability worst-case reliability measure

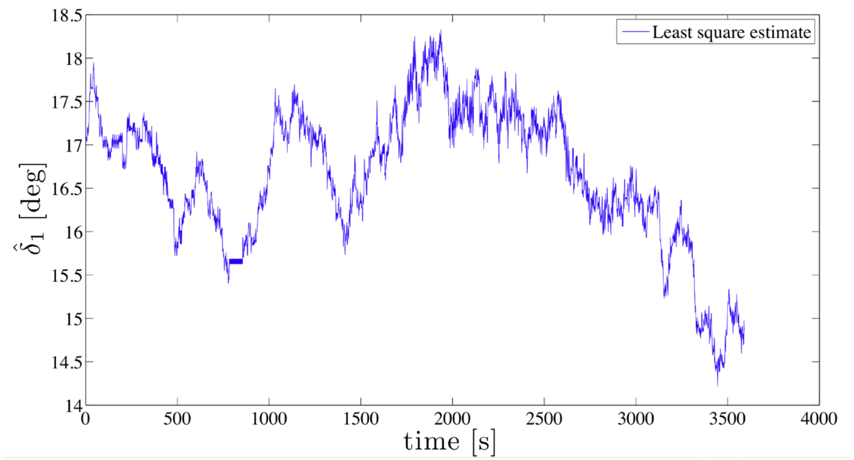
$$H^s = \max_i \{L^s[i]\}, \; i = 1, 2, \cdots, 3600$$

765 kV ***Line Case Study***

- Stability margin analysis
- Date:
  - 09/03/10
- Time horizon:
  - 18:07:12EDT-19:07:12EDT
- $E_1 = E_2 = 765$ kV (assumed)

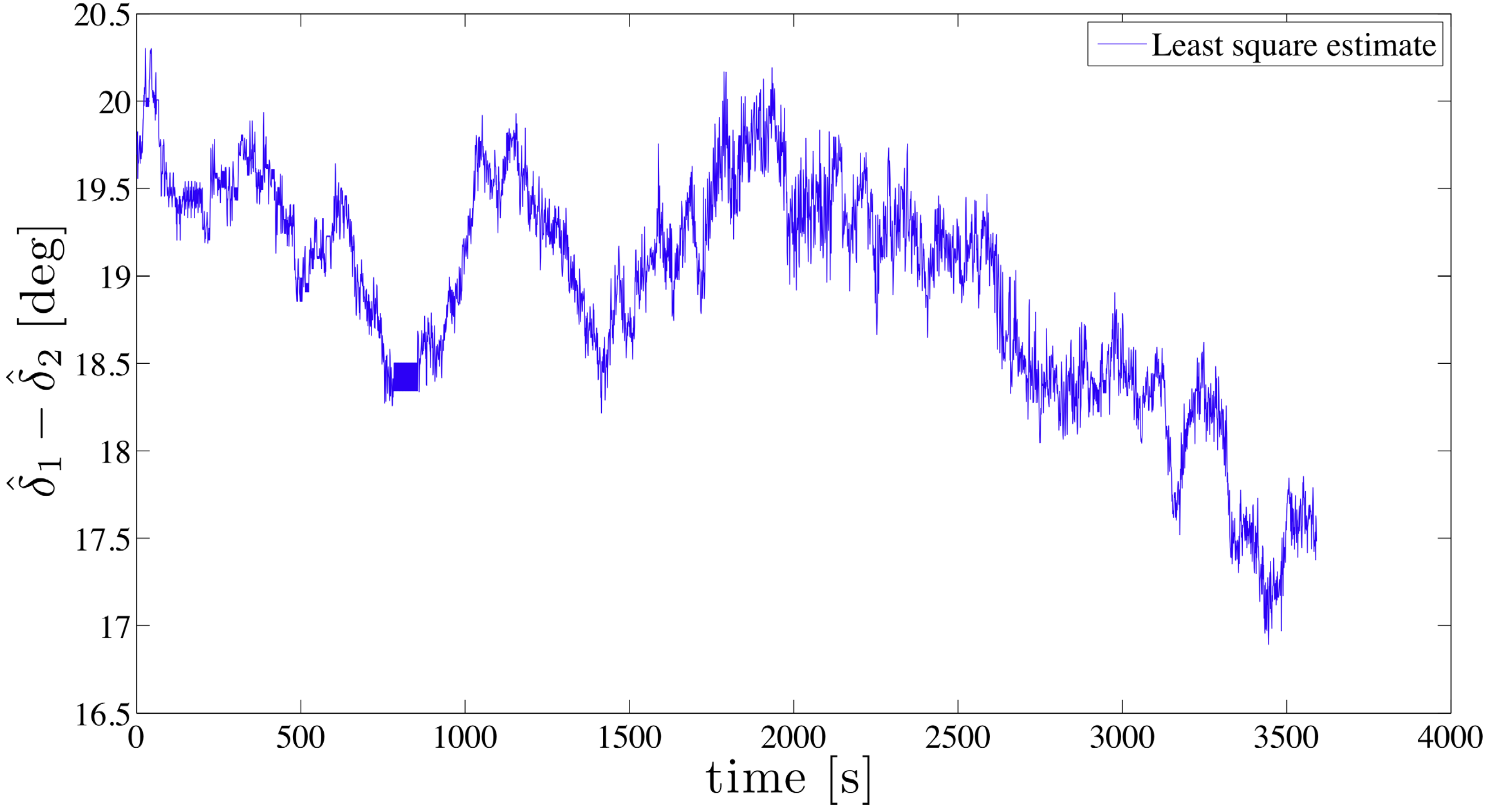






Thevenin parameter estimates for equivalent

Angle across the system measure



Issues

- We know that the Thevenin equivalent changes between samples
- What does a “changing” equivalent mean in the math?
- How many samples do we need to have a well-conditioned problem?
- How can we decide if the problem is well-conditioned?
- Intrusion into the process could impact conditioning.
- What about contingencies (N-1)?
- Is the 45 degrees criteria correct?
- What is the “path” to the bifurcation?
- Is the “path” to the bifurcation important?
- How many lines need to be monitored?
- How do we verify this is correct?
- PMU data quality
- Can we push the computation down to the substation?
- A lot of this is really hard to prove because we do not know the answers!

Cyber threats

- Spoofing of the Global Positioning System (GPS) would inject errors in PMU data and provide the wrong margin to maximum loadability – this is a TCIPG activity

- Hacking into the data could result in ill-conditioned matrices that could not compute equivalents or operational limits – or erroneous equivalents that give the wrong margins.

Conclusions

- PMU data offers the potential to perform operational reliability analysis without extensive model data.
- There are technical issues yet to be clarified and shown.
- There are cyber security issues.
- Issue of contingencies needs to be included.
- PMU data quality needs to be assured.
- Tests for proper conditioning of data and matrices are needed