A Complex Network View of the Grid

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TCIPG

Motivation

- Power grids have grown organically over the past century (naturally random)
  - More balancing options: economic benefits + safety
- Design and analysis of power grids has been based on reference samples and case studies
  - Does not help establishing macroscopic trends
- Can we capture in a model key features of the ensemble?
- Does it give useful insights?

Background

The grid: a system of systems

- A complex system view focuses on how they are "randomly" coupled

Generators,Loads

Transmission Lines

Power systems gear: Switches, Relays,Transformers...

Computers and Sensors (Substations, PLC, Supervisory control)

Market players (supply and demand)

How do power engineers grasp trends?

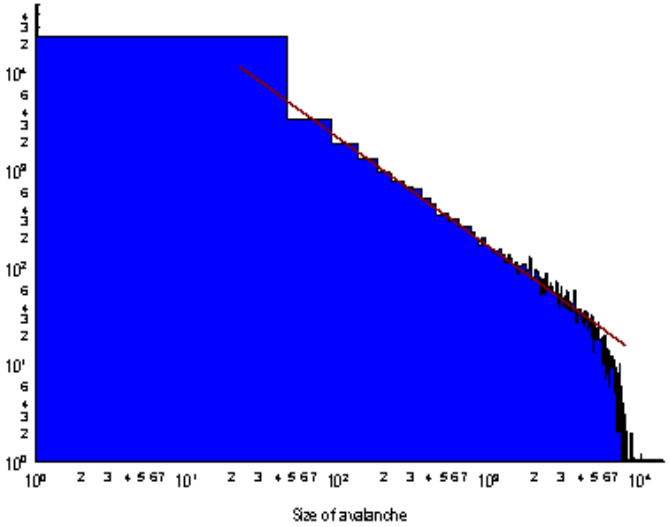
- Most of the literature has used real grids or reference models for testing ideas and gaining insight
  - IEEE 30 57, 118 and 300 bus systems
  - Power systems test case archive
    - http://www.ee.washington.edu/research/pstca/
- Scalable models to grasp macroscopic trends
  - [Parashar and Thorp ’04] ring topology + “continuum model”
  - [Rosas-Casals, Valverde, Solé ’07] tree topology
- The bias is towards deterministic models

Cascading failure models

- Carreras, Newman, Dobson, Lynch…. in a series of papers from ~2002 to present worked on the analysis and modeling of the self-critical behavior of cascading failure

Size of failure exhibits power law scaling behavior in NERC data as well as in models (exponent -1.2 or -1.5)

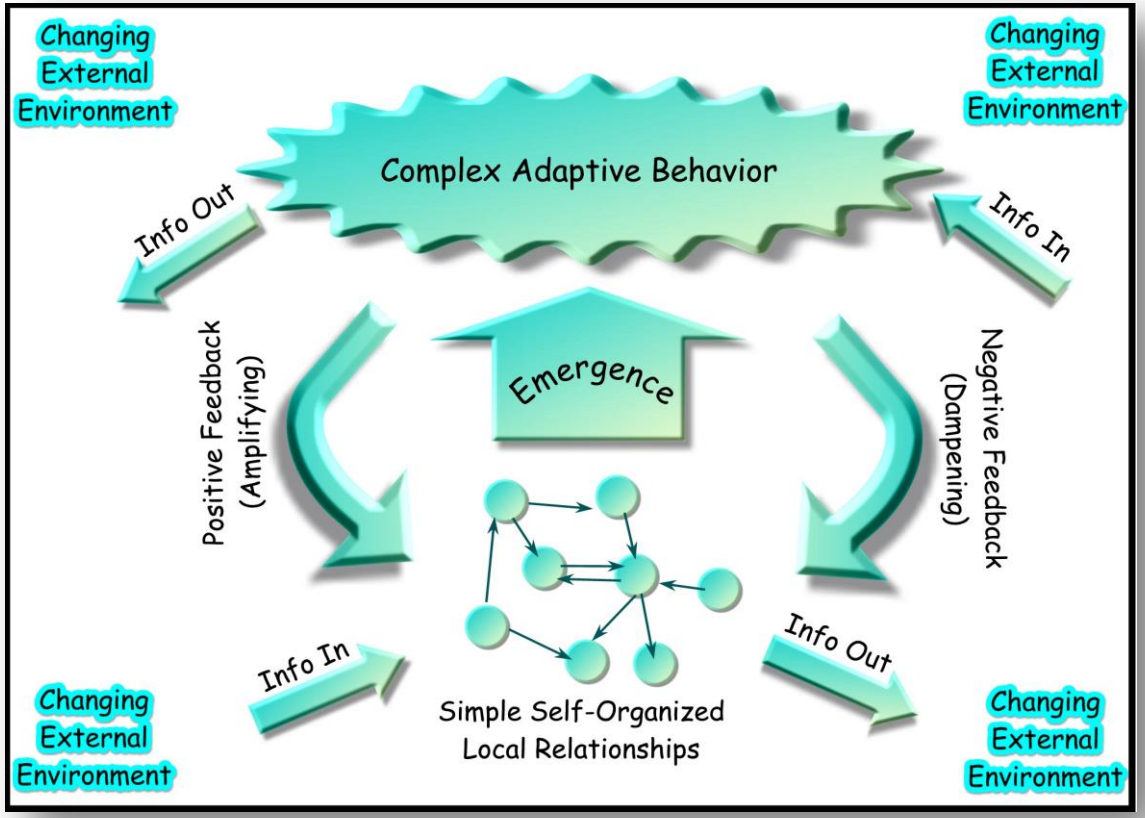
Also in this case test cases are used…



- Why even if we use different test grids we get the same cascading trends….

Complex Systems Theory

- It is a modern branch of (statistical) physics
- Searches the laws that explain the *emergence of macroscopic phenomena*



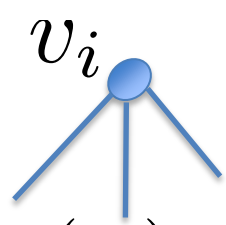
Random graph models

- Uniform choice → Erdős–Rényi Graph
  - G(n,p) one of the possible n(n-1) edges is included with probability p
- In space → Random Geometric Graph
  - G(n,r) Nodes are placed uniformly at random in an unit area and they are connected if their distance is less than the radius r
- Examples of emergent behavior: Phase transition
  - ERG → G(n, 2ln(n)/n) is connected almost surely
  - RGG → G(n, $(\ln(n)/\pi n)^{1/2}$)) is connected almost surely

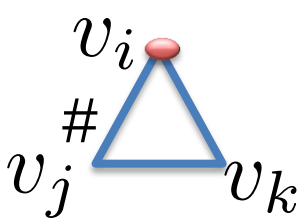
More complex models

- Many real graphs features are inconsistent with such simple behavior

Features examined



$deg(v_i) = 3$



  - Heavy tail degrees and heavy "clustering" (triangles) are frequent in real world graphs…

- Preferential attachment → Barabási–Albert (BA)

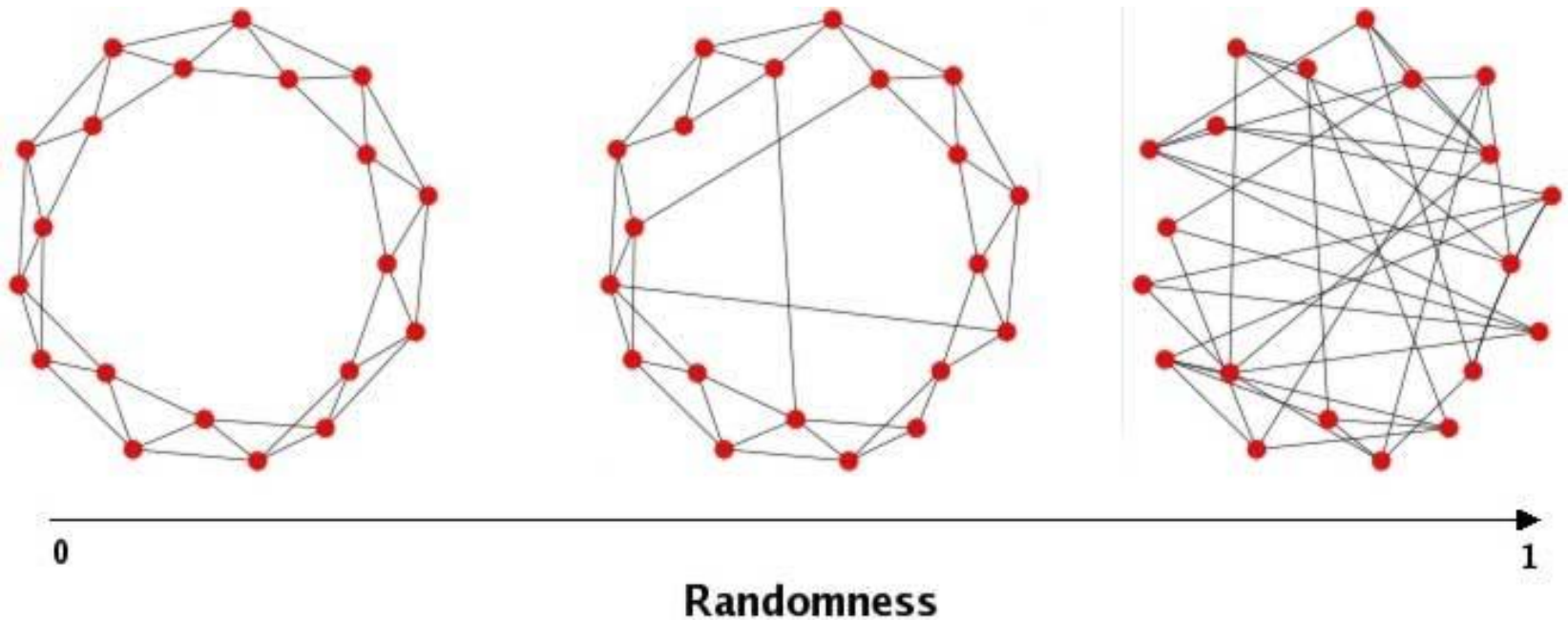
Growth model via prob. of choosing node v_i $\quad p_i = \frac{k_i}{\sum_j k_j}$

  - Degree distribution is a power law (scale free graph)

Small world model

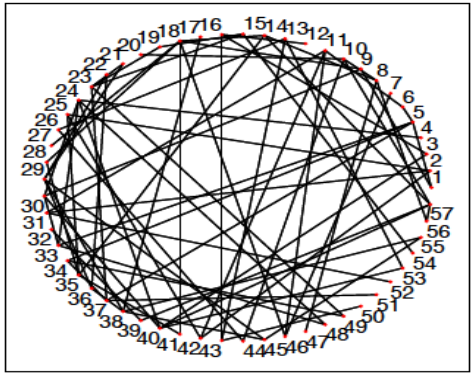
- ‘98 Watts and Strogatz, *Nature*

Deterministic | Limited random re-wiring: Small World | Totally random Erdős–Rényi

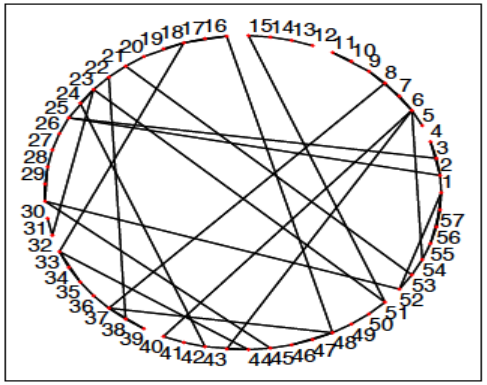


Visual comparison with circular embedding

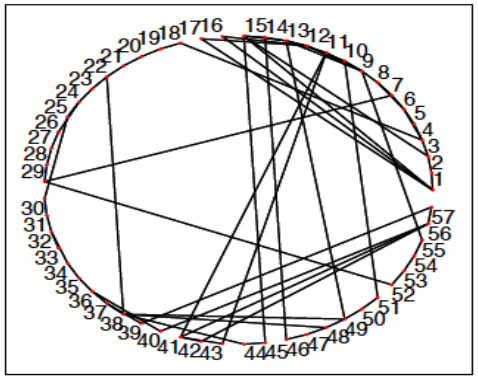
Erdos Reny



Small World



Power network



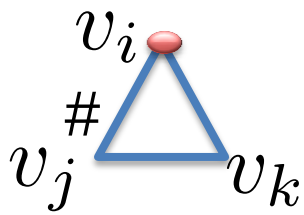
- Watts and Strogatz (eyeballing these graph)
  Conjecture: Power Grids are small world networks
  - Power grid specific →Topological studies: [Newman ’03], [Whitney & Alderson’06][Wang, Rong,’09], Degree distribution: [Albert et al. ‘04],[Rosas-Casals et al. ‘07]

Small world → high clustering coefficient

- high average clustering coefficient of the sample power grid network examined

Definition of clustering coefficient

$$C_i = \frac{2|\{e_{jk}\}|}{k_i(k_i-1)}$$



$$\forall\ (v_j, v_k) \in N_i, e_{jk} \in E$$

$$(N_i = \text{ neighbors of } v_i)$$

$$(E = \text{edges})$$

$$(k_i = \text{degree of } i)$$

Erdos Renyi

Grid	$C(G)$	$C(R)$
IEEE-30	0.2348	0.094253
IEEE-57	0.1222	0.048872
IEEE-118	0.1651	0.025931
IEEE-300	0.0856	0.009119
NYISO-2935	0.2134	0.001525
WSCC-4941	0.0801	0.000540

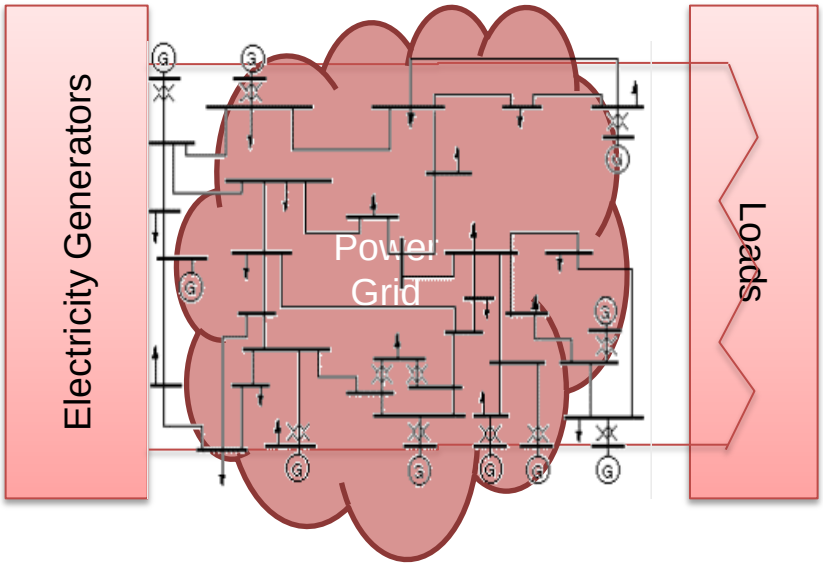
Can this approach provide insights?

- Criticism: the results are not related with the physical laws that govern the grid

We first analyze more carefully several test topologies and study all the relevant statistics and then we revisit this question

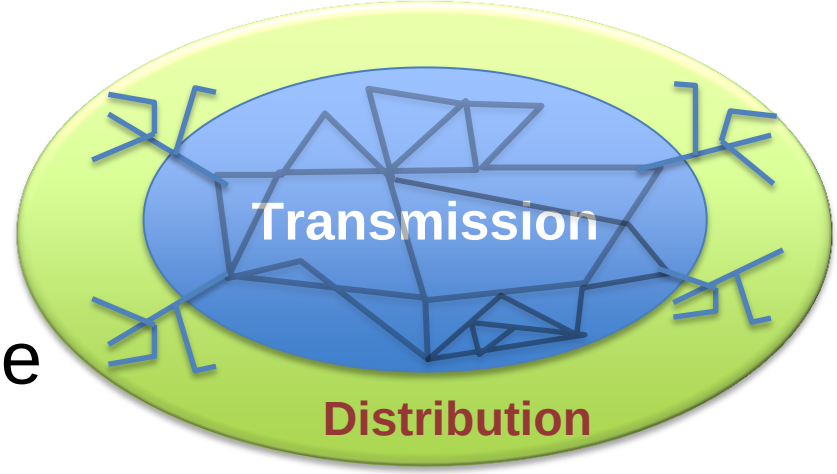
What we model

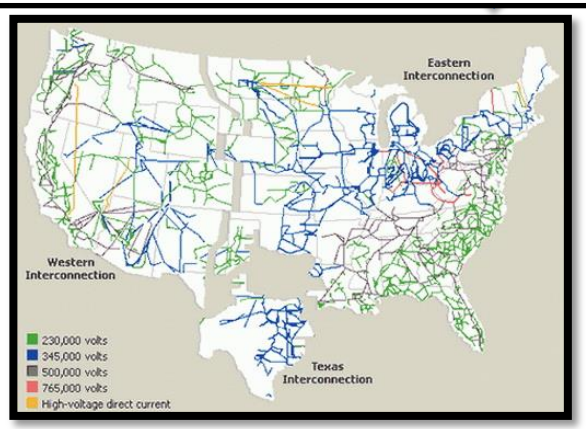
- Topological and electrical characteristics of the transmission grid
- The scaling trends observed in considering wider portions of the grid
- The statistical properties of the grid admittance matrix are what matters, since it expresses how electric power is constrained to flow

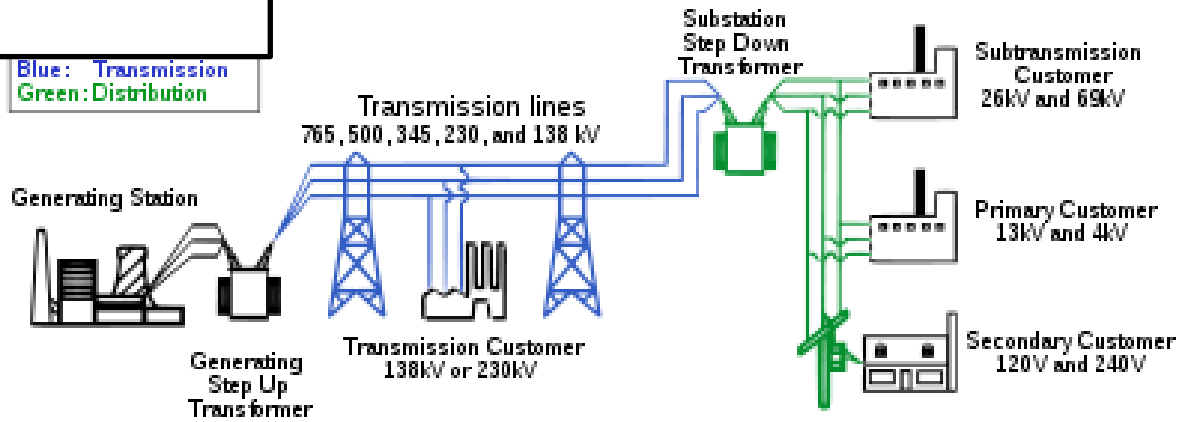


The grid transmission lines

- Our data are for the High Voltage/Transmission section
- Also one data point for Medium Voltage Distribution
- Leave out the distribution network (typically radial)







Admittance matrix and the graph topology

- Line-Node Incidence Matrix (M x N):

$$\text{Line } i \text{ connected to node } j \rightarrow A_{i,j} = 1,\ A_{j,i} = -1$$
$$\text{else } A_{i,j} = A_{j,i} = 0$$

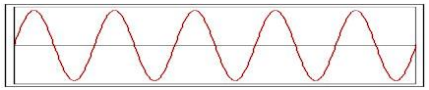
- Admittance matrix

$$Y = A^T diag(y_1, \ldots, y_M) A$$

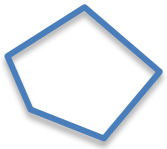
- Observation: Y is a weighted graph Laplacian
  - complex weights given by the admittances of the lines

The laws for the grid

- Voltage, Currents, Powers → narrow spectrum

AC ~ 60-50 Hz 

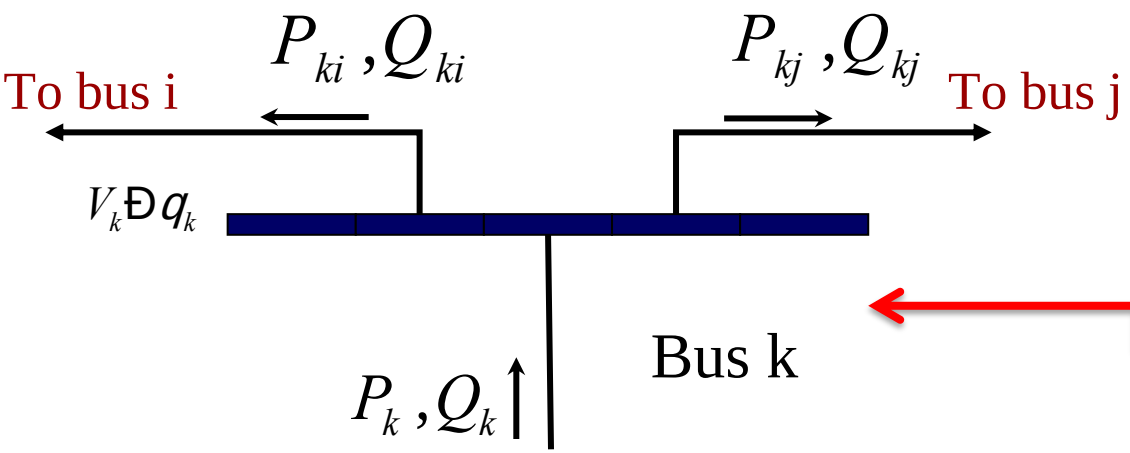
- Electrical transient dynamics → unimportant
  - Circuit laws replaced with algebraic equations (frequency) relating "*phasors*" (complex numbers whose phase and amplitude match the AC signal V and I)
- Kirchhoff's Voltage/Current laws (KVL-KCL)

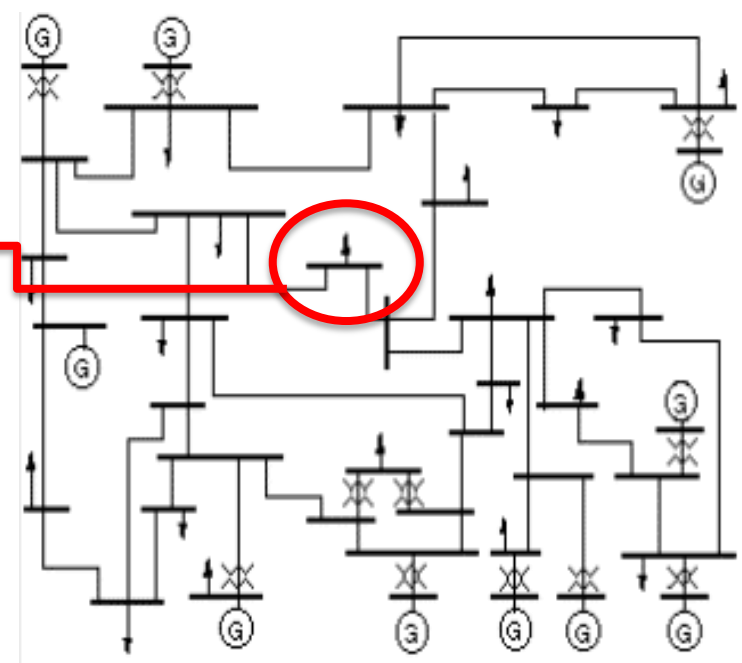


$$\sum_{i \in \text{Circuit}} V_i = 0, \qquad \sum_{i \in \text{Node}} I_i = 0$$

- Ohm's law $$V_i = Z_i I_i$$

Relationship with power: The balance equations





The properties of the topology and the random admittance of the lines end up shaping how the power flows through the power flow equations

Power Injection = Losses Admittance matrix

$$P_i + jQ_i = V_i I_i^* = V_i \sum_{j \in B} Y_{ij}^* V_j^*$$

Random Grids Characteristics

Degree distribution

- [Albert et al. ’04,Rosas-Casals’07] Geometric PDF
- Way to highlight:

  Probability Generating Function (PGF)

  - For a mixture model

$$G_k(z) = G_{k_1}(z) \dots G_{k_p}(z)$$

Our analysis result

1. The degree distribution is a mixture of a truncated exponential and finite support random variable
2. The average degree vs. N is O(1)

Why the PGF?

- A finite support Probability Mass Function (PMF) is a finite order polynomial
  - We should see ‘zeros’ in the PGF

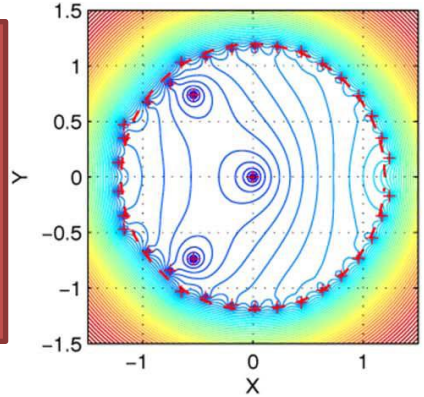
$$G_{\mathcal{D}}(z) = p_0 + p_1 z + \ldots + p_{k_t} z^{k_t}$$

- A purely geometric random variable is the reciprocal of a first order polynomial → ‘pole’
  - Impossible to observe, in practice a ‘clipped’ version

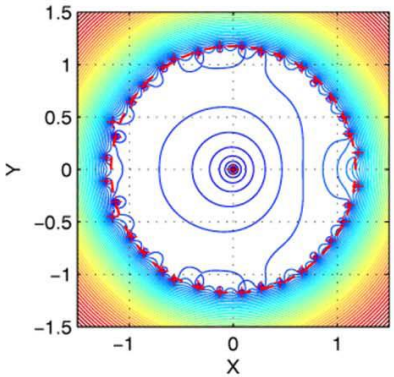
$$G_{\mathcal{G}}(z) \propto \frac{1 - [z(1-p)]^{k_{\max}+1}}{1 - (1-p)z}$$

PGF NYSO data

(a) All buses
(b) Gen buses
(c) Load buses.
(d) Connection buses.
(e) Gen+Load buses.
The zeros are red '+'



(a)



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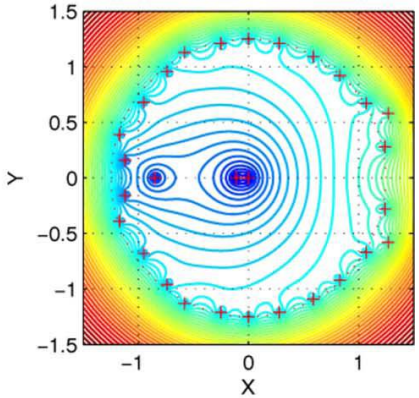

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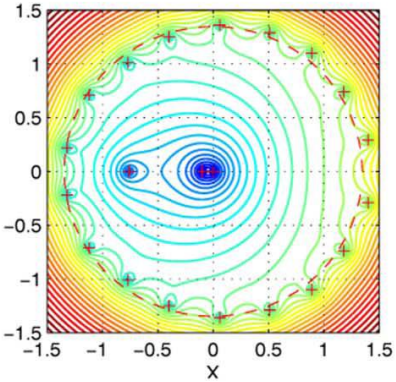

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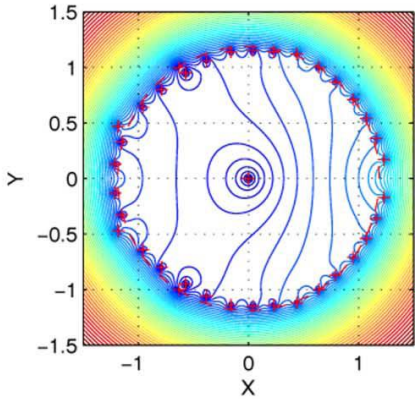
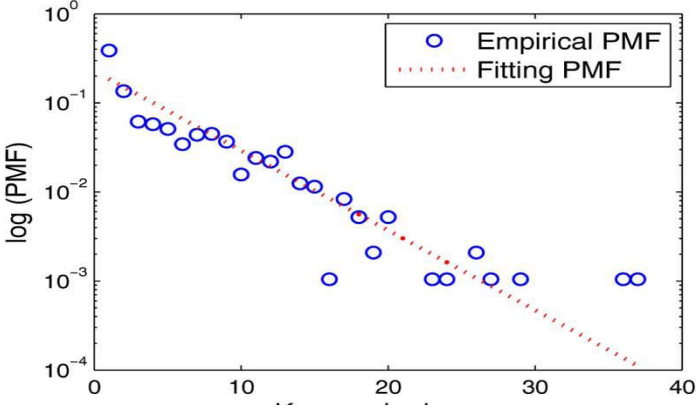
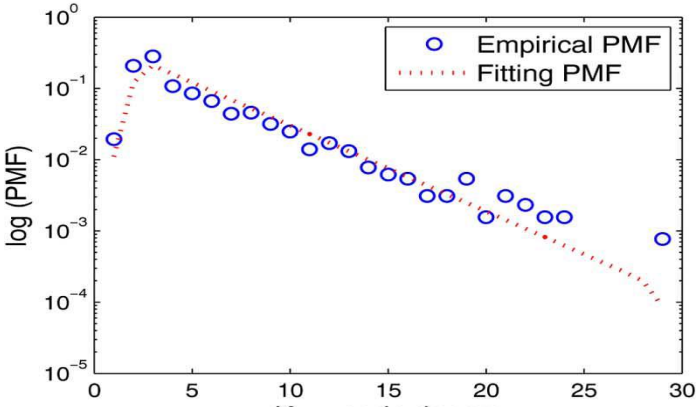


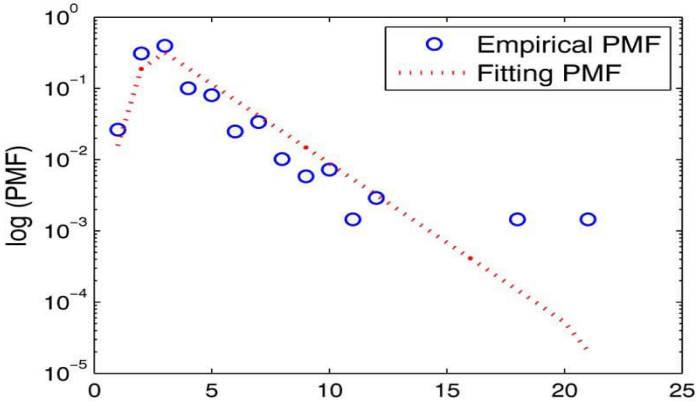
(e)



Degree of Generator buses



Degree of Load buses



Degree of Connection buses

WSCC versus NYSO degree distribution

$$G_k(z) = G_{\mathcal{D}}(z) G_{\mathcal{G}}(z)$$

ESTIMATE COEFFICIENTS OF THE TRUNCATED GEOMETRIC AND THE IRREGULAR DISCRETE FOR THE NODE DEGREES IN THE NYISO AND WSCC SYSTEM

node groups	$\max(\underline{k})$	p	k_{max}	k_t	$\{p_1, p_2, \cdots, p_{k_t}\}$
All	37	0.2269	34	3	0.4875, 0.2700, 0.2425
Gen	37	0.1863	36	1	1.000
Load	29	0.2423	26	3	0.0455, 0.4675, 0.4870
Conn	21	0.4006	18	3	0.0393, 0.4442, 0.5165
Gen+Load	37	0.2227	34	3	0.4645, 0.3385, 0.1970
All-WSCC	19	0.4084	16	3	0.3545, 0.4499, 0.1956

Small World conjecture

- Some evidence contradicting it
  - For a SW network with N nodes, to guarantee with high probability a connected network (no isolated component) the scaling laws for the average degree <*k*> >> *log N*
  - The average degree in power grids is ~ constant (3-4)

N: Number of nodes
m: number of lines
-
<k> Average Degree
<l> Average shortest path length
ρ Pearson Coefficient
r{k>k} Ratio of nodes with largest nodal degree

TOPOLOGICAL CHARACTERISTICS OF REAL-WORLD POWER NETWORKS

	(N, m)	$\langle l \rangle$	$\langle k \rangle$	ρ	$r\{k > \bar{k}\}$
IEEE-30	(30,41)	3.31	2.73	-0.0868	0.2333
IEEE-57	(57,78)	4.95	2.74	0.2432	0.2105
IEEE-118	(118,179)	6.31	3.03	-0.1526	0.3051
IEEE-300	(300, 409)	9.94	2.73	-0.2206	0.2367
NYISO	(2935,6567)	16.43	4.47	0.4593	0.1428
WSCC	(4941, 6594)	18.70	2.67	0.0035	0.2022

Average shortest path

- Observation: $< l > \approx 3 \log_{10}(N)$

N: Number of nodes
m: number of lines
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TOPOLOGICAL CHARACTERISTICS OF REAL-WORLD POWER NETWORKS

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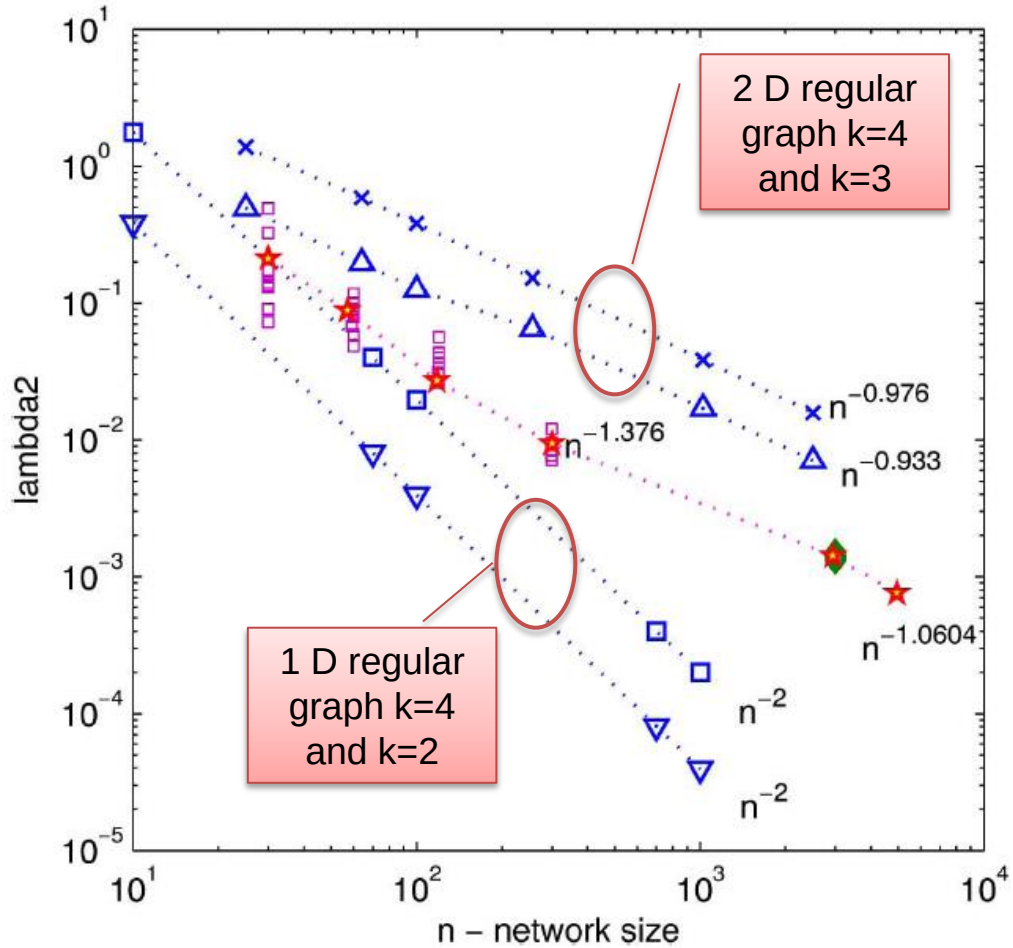
- Not bad to overlay communications with the lines – relatively short distance

Algebraic connectivity

- Graph Laplacian second smallest eigenvalue $\lambda_2(L)$
- Values shown in I

	$\lambda_2(L)$
IEEE-30	0.21213
IEEE-57	0.088223
IEEE-118	0.027132
IEEE-300	0.0093838
NYISO-2935	0.0014215
WSCC-4941	0.00075921



Significance of algebraic connectivity

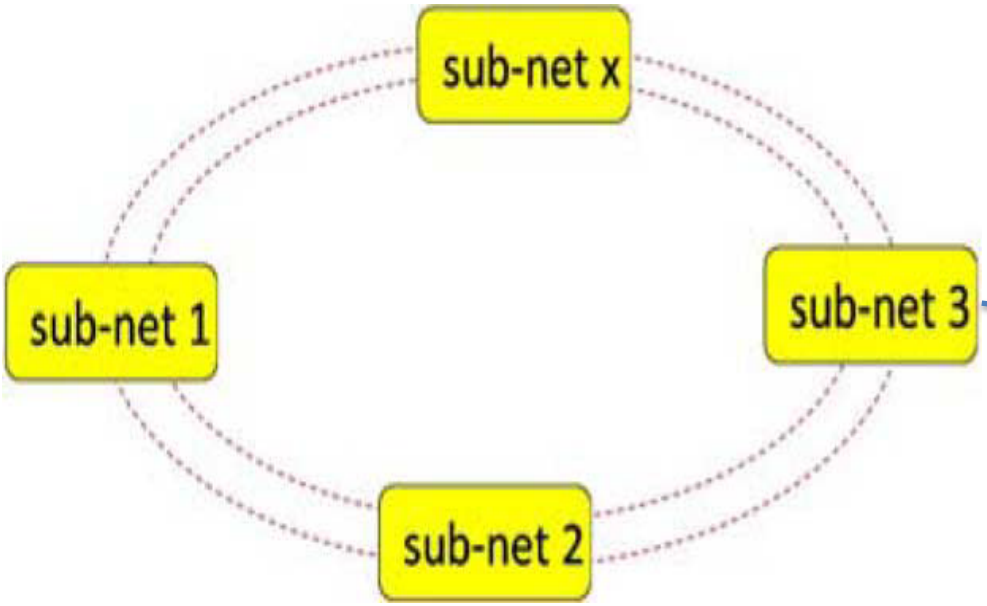
- The nullity (dimension of the kernel) of the graph Laplacian indicates how many connected components are in the graph
  - The graph is connected if and only if

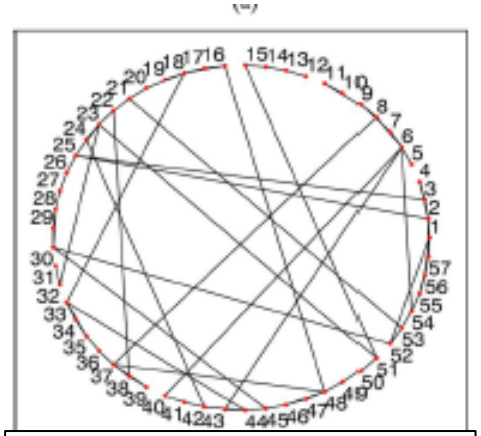
$$\lambda_2(L) > 0$$

- Mixing time
  - Normalized L transition probability matrix of a Markov chain → large algebraic connectivity, fast convergence to uniform stationary distribution
- Heat Diffusion
  - The Graph Laplacian is the discrete equivalent of the Laplace Beltrami operator → large algebraic connectivity, fast temperature equilibrium

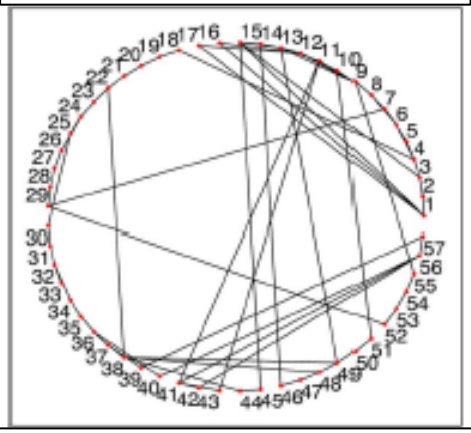
Plausible topology

- The model that matches this trend is what we call Nested-Small-world graph
  - IEEE → SW subnet 30; NYSO & WSCC → SW sub-net 300





SW: independent rewiring



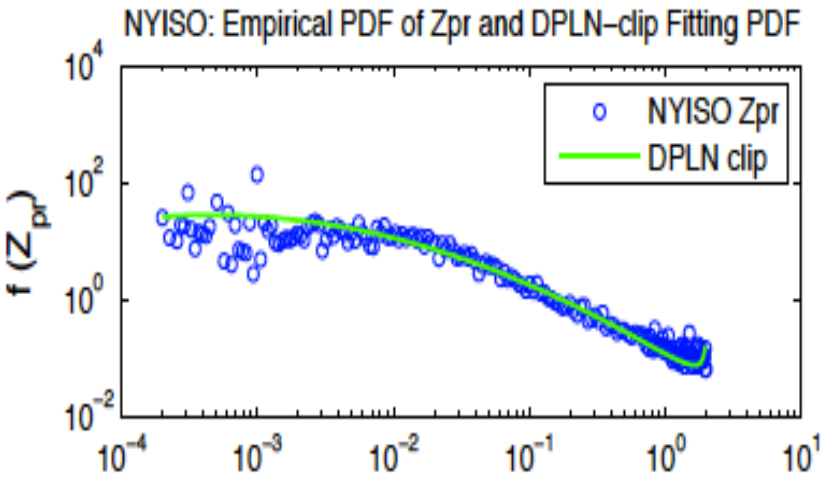
IEEE 300: Correlated rewiring

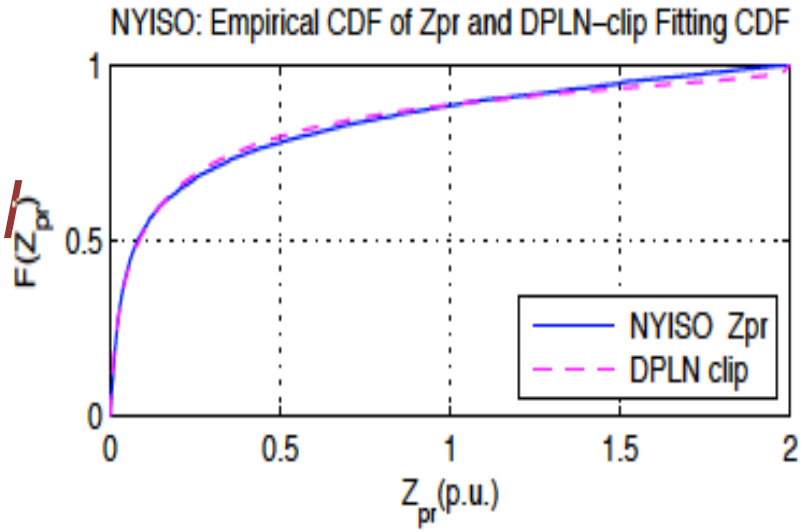
Impedance distribution

- Absolute values of the impedances

$$Z_{pr} = R + jX \approx jX$$

- Prevailingly *<u>heavy tailed distributions</u>*
- NYSO best fit → *clipped Double Pareto Log-normal*
  - Did not pass KS test but was the closest to pass it





Distributions comparison

DISTRIBUTION FITTING FOR LINE IMPEDANCES

System	Fitting Distribution	ML Parameter Estimates (*alph*=0.05)
IEEE-30	$\Gamma(x\|a,b)$	$a = 2.14687$ $b = 0.10191$
IEEE-118	$\Gamma(x\|a,b)$	$a = 1.88734$ $b = 0.05856$
IEEE-57	$gp(x\|k,\sigma,\theta)$	$k = 0.33941$ $\sigma = 0.16963$ $\theta = 0.16963,$
IEEE-300	$gp(x\|k,\sigma,\theta)$	$k = 0.45019$ $\sigma = 0.07486$ $\theta = 0.00046,$
NYISO-2935	$logn_{clip}(x\|\mu,\sigma,Z_{\max})$	$\mu = -2.37419$ $\sigma = 2.08285$ $Z_{max} = 1.9977$
	$dPlN_{clip}(x\|\alpha,\beta,\mu,\sigma,Z_{\max})$	$\alpha = 44.25000$ $\beta = 44.30000$ $\mu = -2.37420$ $\sigma = 2.082600$ $Z_{max} = 1.9977$

Gamma:

$$\Gamma(x \mid a,b) = \frac{1}{b^a\Gamma(a)} x^{a-1} e^{x/b}.$$

Generalized Pareto (GP):

$$gp(x \mid k,\sigma,\theta) = \left(\frac{1}{\sigma}\right)\left(1 + k\frac{(x-\theta)}{\sigma}\right)^{-1-(1/k)}.$$

Lognormal:

$$\log n(x \mid \mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\log x - u)^2}{2\sigma^2}}.$$

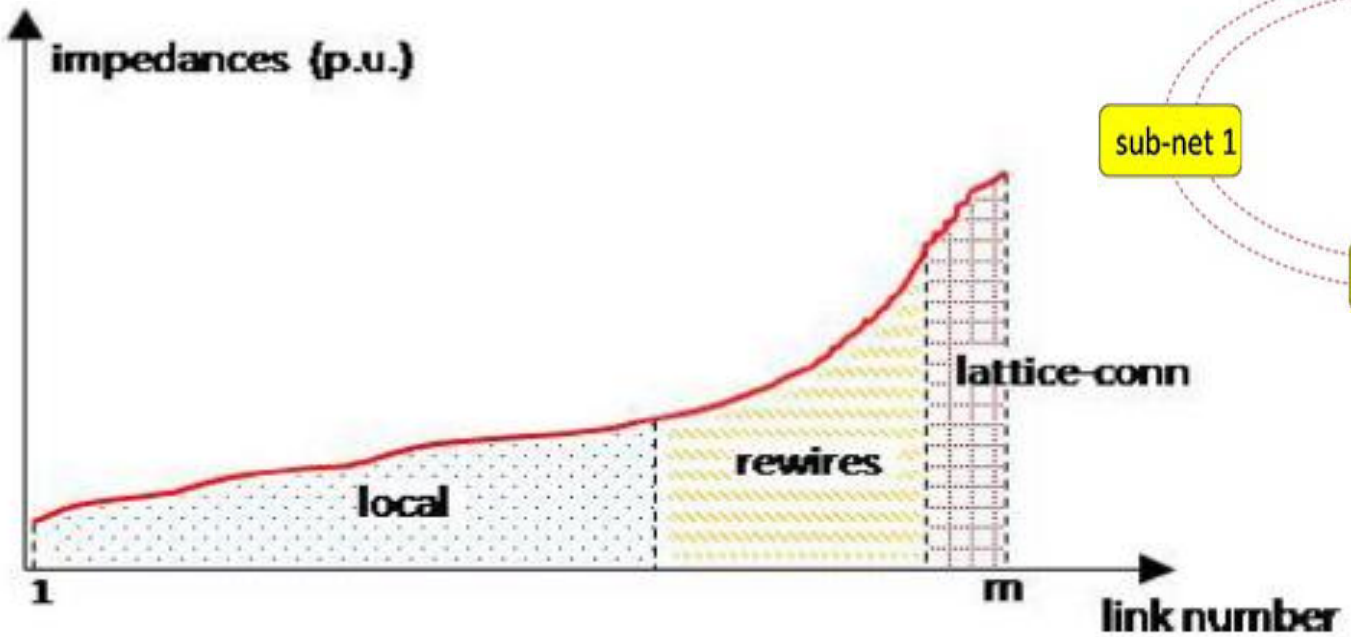
DPLN:

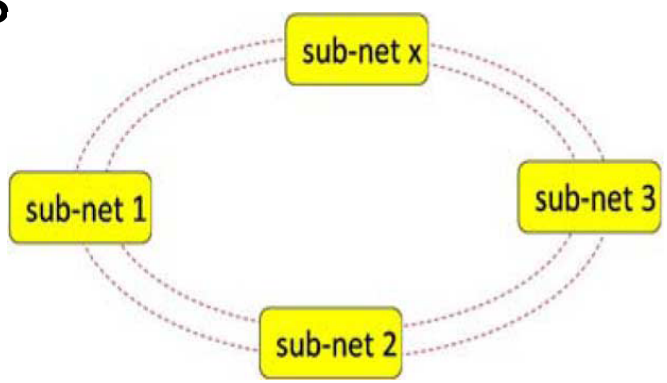
$$\begin{aligned}\mathrm{DPLN}(x|\alpha,\beta,\mu,\sigma) &= \frac{\alpha\beta}{\alpha+\beta}\Bigg[A(\alpha,\mu,\sigma)x^{-\alpha-1}\Phi\left(\frac{\log x - \mu - \alpha\sigma^2}{\sigma}\right) \\ &\quad + A(-\beta,\mu,\sigma)x^{\beta-1}\Phi^c\left(\frac{\log x - \mu + \beta\sigma^2}{\sigma}\right)\Bigg].\end{aligned}$$

where $A(\theta,\mu,\sigma) = e^{(\theta\mu+\theta^2\sigma^2/2)}$.

Impedance attribution

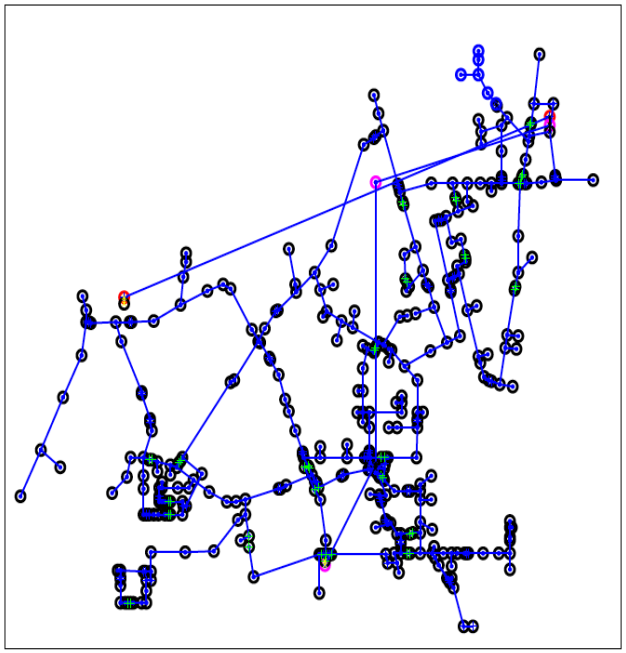
- Impedance grows with distance
- Conjecture: local → short; rewires → medium; lattice connections → long lines





396-node Medium Voltage distribution network

- US distribution utility
  - The power supply from the 115 kV-34.5 kV step-down substation.
  - Most nodes or buses in the network are 12.47 kV (>95%), and only a small number of them are 34.5 kV or 4.8 kV.



	(N, m)	$\langle k \rangle$	$\langle l \rangle$	ρ	$\lambda_2(L)$	$C(G)$
IEEE-300	(300, 409)	2.73	9.94	-0.2206	0.0094	0.0856
WSCC	(4941, 6594)	2.67	18.70	0.0035	0.00076	0.0801
396-node MV-Distr	(396, 420)	2.12	21.10	-0.2257	0.00030	0

Insights

Vulnerability studies

- Fraction of nodes removal before breakdown
  - R. Cohen, K. Erez, D. ben-Avraham, S. Havlin '00 provided an analysis that requires the degree distribution

If for the spanning components all edges connect nodes with average degree 2 the network is at the critical transition

$$\kappa = < k_i | i \to j > = \frac{< k^2 >}{< k >} = 2$$

  - Removing edges with probability f^{rand}

$$\kappa = \kappa_0 (1 - f^{rand}) + f^{rand} = 2$$

$$\to f^{rand} = 1 - \frac{1}{\kappa_0 - 1}$$

Vulnerability studies

- Selective removal rate before breakdown
  - Sole, Rosas-Casals, Corominas-Murtra, and Valverde '07
    - Start from the nodes with highest degree first and remove edges with probability f^{sel}
- For a purely geometric random degree distribution

$$f^{rand} \approx \left(1 - \ln f^{sel}\right) f^{sel} = 1 - \frac{1}{\kappa_0 - 1}$$

Accounting for true degree distribution

- [Wang, Scaglione, Thomas '09]

$$f^{rand} \approx \left(1 - \ln \frac{f^{sel}}{r_0}\right) f^{sel} = 1 - \frac{1}{\kappa_0 - 1}$$

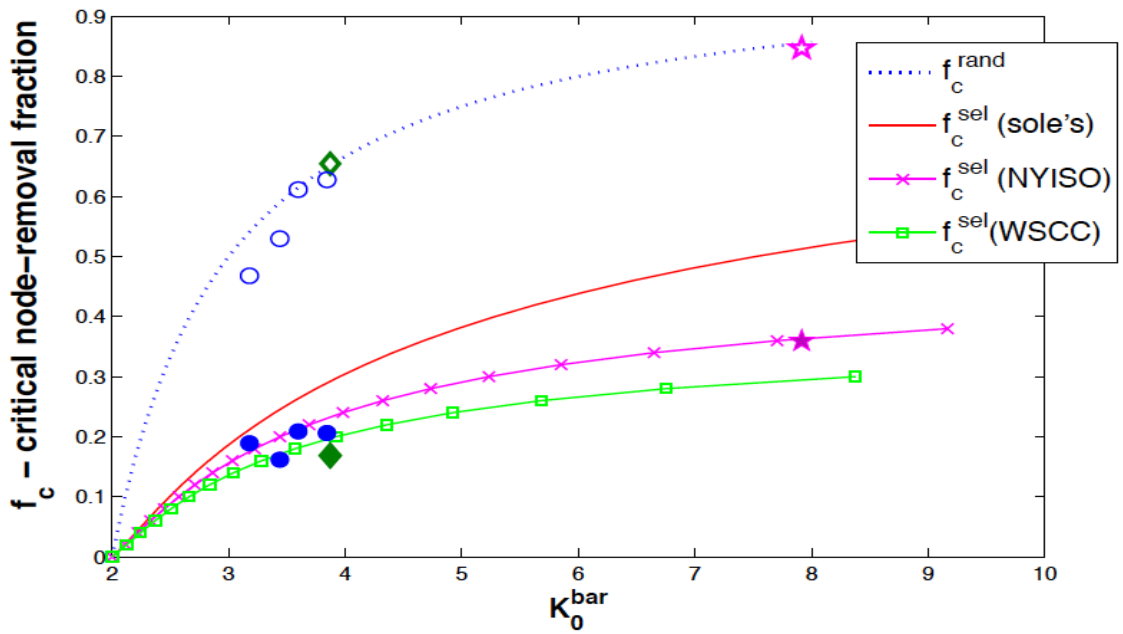
The Theoretical versus the Empirical Critical Breakdown Thresholds

$$r_{NYISO} = 1.4074$$

$$r_{WSCC} = 1.9690$$

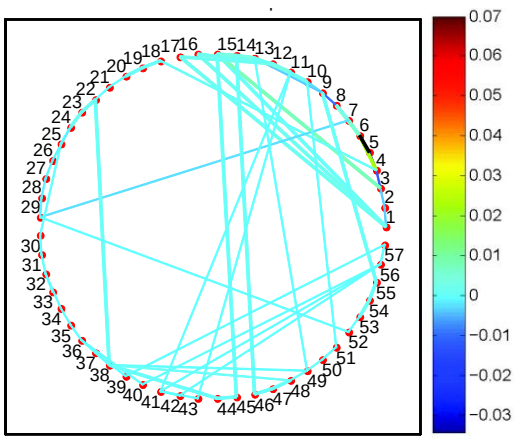
IEEE (circles),
***WSCC*(diamond),**
NYISO (star)
Hollow - f_c^{rand}

Filled - f_c^{sel}

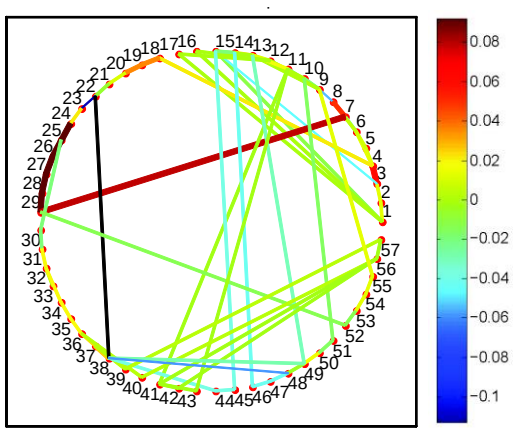


Cascading failures?

- A number of papers argued that congestion in the grid transfers through near neighbors
- Topology is all you need to study this but
  - Kirkoff law could have a similar effect, but voltage law and Ohm's law make a significant difference



local line (5-6) tripped



rewire link (22-28) tripped

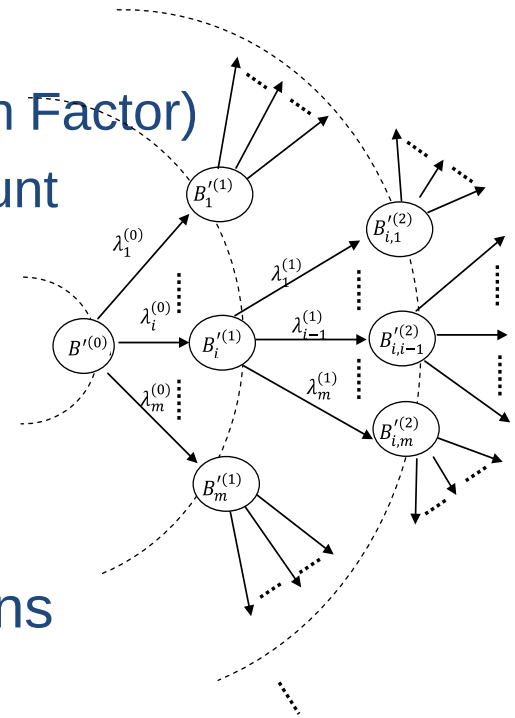
The flow redistribution does not concentrate on shortest path, nor does it distributed according to node degrees.

Where does this leave us?

AC Power Flow

$$\mathbf{P}_G - \mathbf{P}_L = diag(\mathbf{V})\mathbf{Y}^*\mathbf{V}^*$$

- Cascading failures so far are numerical models
  I. Take a specific operating point, Fail a line
  II. Calculate new connectivity
  III. Calculate new flows (Line Outage Distribution Factor)
  IV. (Optional) take other failure models into account
  V. (Optional) Optimum generation re-dispatch
  VI. Trip all violating lines
  VII. Stop if no violations, otherwise go II.



- Typically use DC power flow
  - no averaging over load and generation conditions
  - no load and generator dynamics

Geometrical insights from AC to DC Power flow

- Admittance matrix $\mathbf{Y} = \mathbf{G} + \mathrm{i}\mathbf{B}'$

conductance

susceptance

- Susceptance >> Conductance
- Small angle difference $|\theta_i - \theta_j|, V_i \approx 1$
- DC Power Flow Model approximaion

$$\mathbf{P} = \mathbf{B}\boldsymbol{\theta}, \quad \mathbf{B} = -\mathbf{B}' \text{ with shunt removed}$$

Power injections

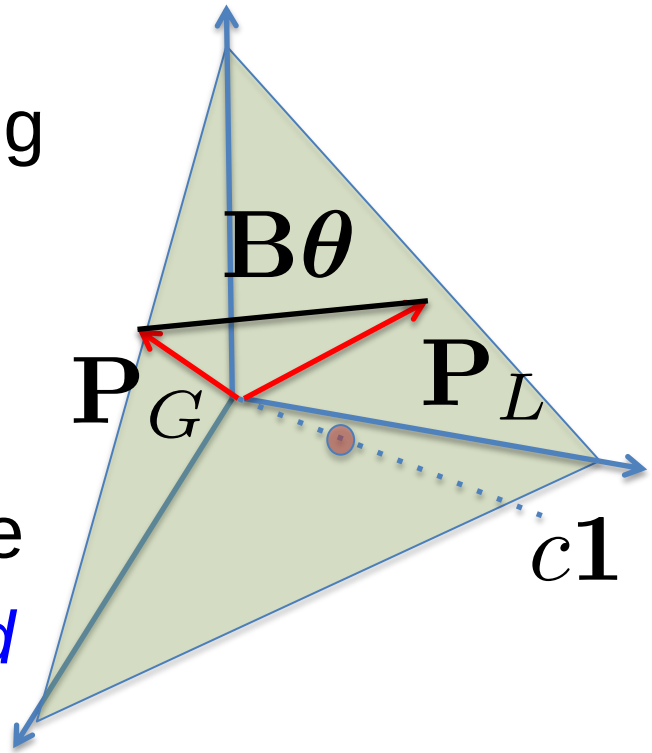
Phase angle

Impact on Power Injections

- The operating condition is the specific load and generation setting

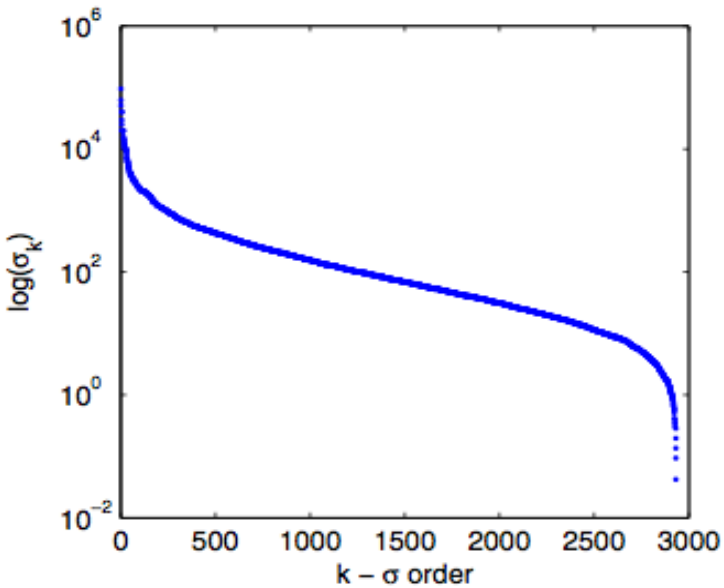
$$\mathbf{B}\boldsymbol{\theta} = \mathbf{P}_G - \mathbf{P}_L$$

- The difference P_G-P_l is confined approximately in a linear subspace
- *The impact of the grid weights and topology is to shape the subspace where the load and generation balance each other*

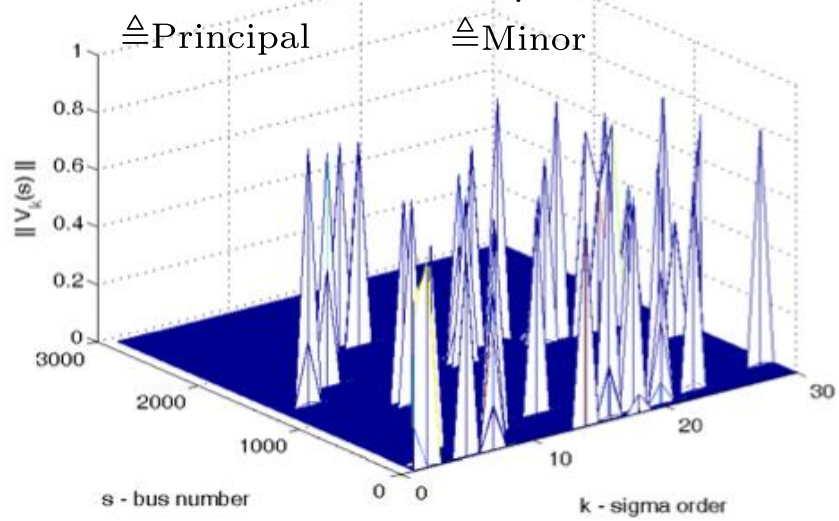


Sparse principal eigenvectors

- We have found that the $\mathbf{Y} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathbf{H}}$ has sparse eigenvalues with sparse principal components



$$\mathbf{U} = [\underbrace{\mathbf{u}_1, \cdots, \mathbf{u}_K}_{\triangleq \text{Principal}}, \underbrace{\mathbf{u}_{K+1}, \cdots, \mathbf{u}_N}_{\triangleq \text{Minor}}]$$



It is a form of “electrical” centrality similar to eigenvalue centrality

Impact on Power Injections

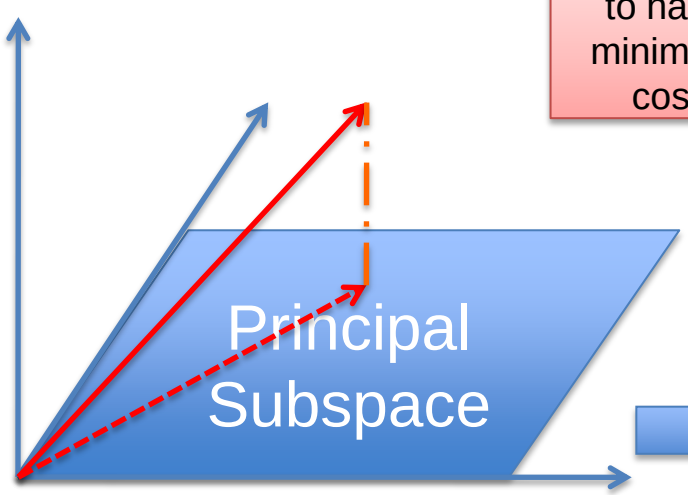
- Low rank approximation $\mathbf{B} = \sum_{k=1}^{K} \lambda_k \mathbf{u}_k \mathbf{u}_k^T + \mathcal{O}(\lambda_{K+1})$

$$\mathbf{B}\boldsymbol{\theta} = \mathbf{P}_G - \mathbf{P}_L$$

$$\sum_{k=1}^{K} \rho_k \lambda_k \mathbf{u}_k \approx \mathbf{P}_G - \mathbf{P}_L, \quad \rho_k = \mathbf{u}_k^T \boldsymbol{\theta}.$$

Dispatched to have minimum cost

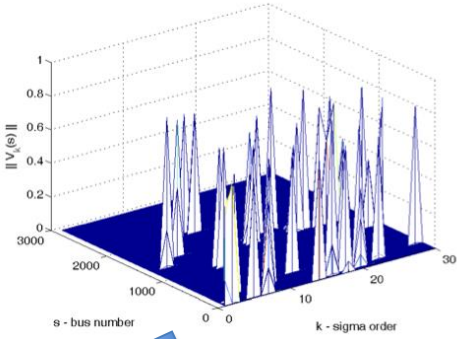
The balance constraint in the Optimal Power Flow Economic dispatch will tend to line up the injection with the principal subspace



Load fluctuation → OPF generation adjustment

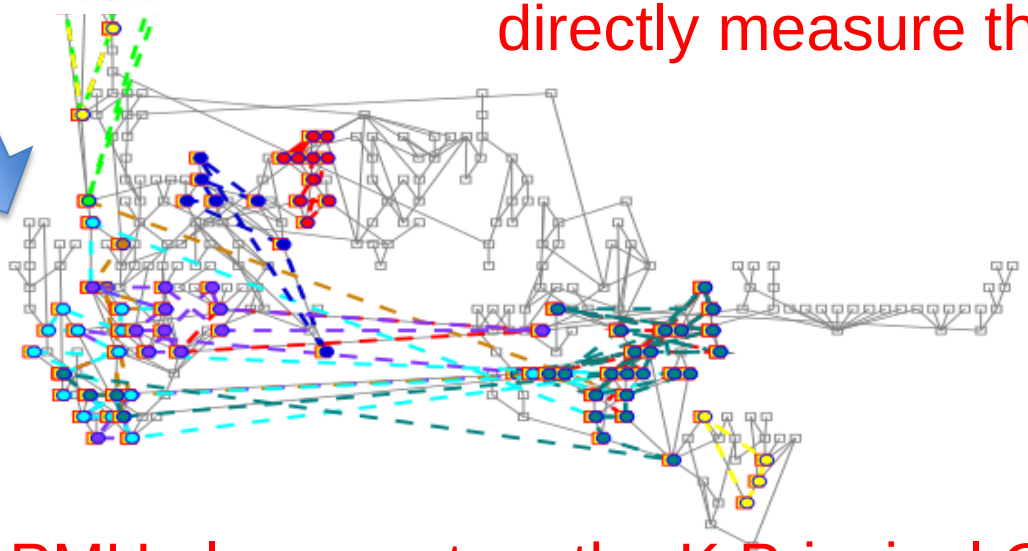
The sensitivity analysis suggests that greatest variations are in the least significant subspace component

Robust state estimation



$$\mathrm{Clique}_k(\gamma) = \left\{ i : [\mathbf{u}_k] \geq \frac{\gamma}{\sqrt{N}} \right\}$$

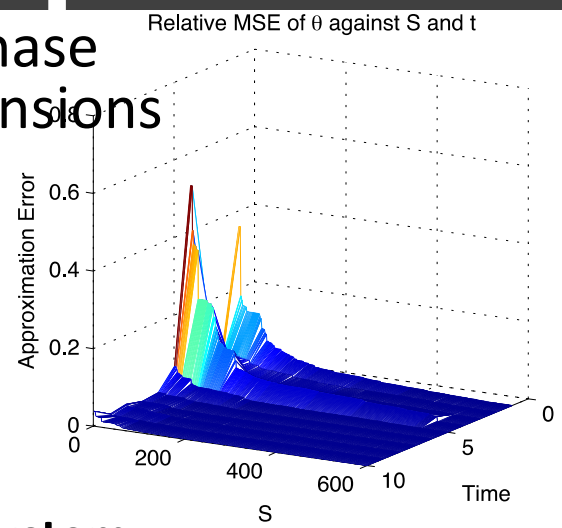
Phasor Measurement Units – directly measure the state V,θ

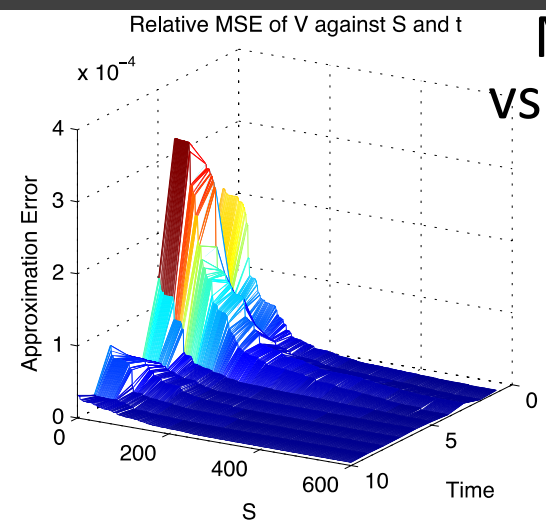


PMU placement on the K <u>Principal Cliques</u> best for accuracy and for stabilizing hybrid State Estimation

Power grid states are compressible

MSE of phase
vs # of dimensions

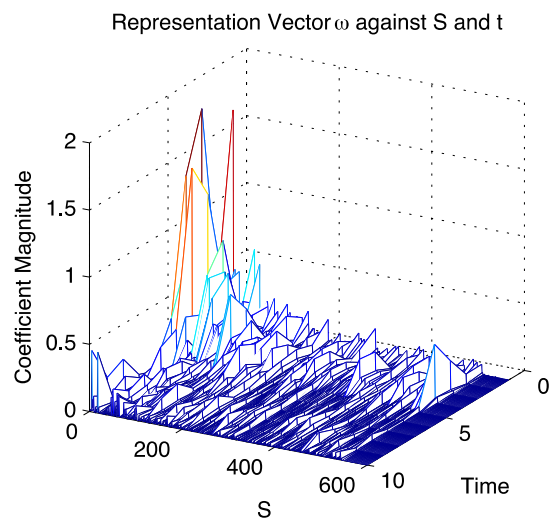


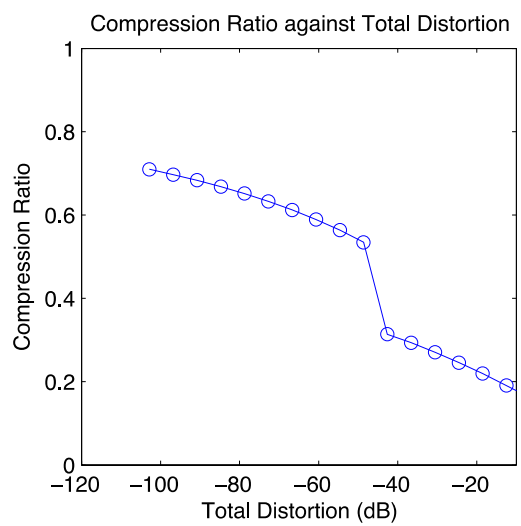


MSE of voltage
vs # of dimensions

IEEE-300 bus system

10 snapshots

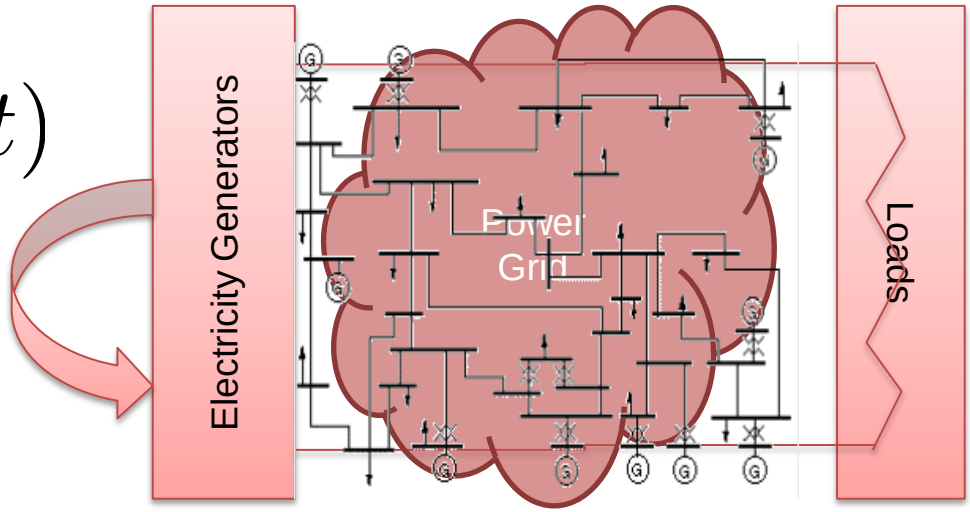




What can be done further?

- the grid does not represent near neighbor exchanges that are typically considered in complex system theory
- We are stuck with numerical models for now

$\mathbf{P}_G(t)$



$\mathbf{P}_L(t)$

Stochastic process

Conclusions

- The admittance matrix of power grids has peculiar features that follow clear statistical trends
- The analysis can help grasping some macroscopic phenomena
- Nevertheless so far cascading failures are only studied through numerical procedures
- The interaction between the load and generators degrees of freedom and the constraint placed by the grid are still there to find

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1) *Zhifang Wang, Anna Scaglione, and Robert J. Thomas* “Generating Statistically Correct Random Topologies for Testing Smart Grid Communication and Control Networks”, IEEE Transactions on Smart Grid, Vol. 1, No. 1. (June 2010), pp. 28-39.
2) *Zhifang Wang, Anna Scaglione, and Robert J. Thomas*, “The Node Degree Distribution in Power Grid and Its Topology Robustness under Random and Selective Node Removals” IEEE International Workshop on Smart Grid Communications, Cape Town, South Africa, May 2010.
3) Zhifang Wang; Scaglione, A.; Thomas, R.J.; , "Compressing Electrical Power Grids," *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on* , vol., no., pp.13-18, 4-6 Oct. 2010
4) Zhifang Wang; Scaglione, A.; Thomas, R.J.; , "Electrical centrality measures for electric power grid vulnerability analysis," *Decision and Control (CDC), 2010 49th IEEE Conference on* , vol., no., pp.5792-5797, 15-17 Dec. 2010
5) Galli, S.; Scaglione, A.; Zhifang Wang; , "For the Grid and Through the Grid: The Role of Power Line Communications in the Smart Grid," *Proceedings of the IEEE* , vol.99, no.6, pp.998-1027, June 2011